Estimating High-Frequency Based (Co-) Variances:

A Unified Approach*

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Abstract

We propose a unified framework for estimating integrated variances and covariances based on simple OLS regressions, allowing for a general market microstructure noise specification. We show that our estimators can outperform in terms of the root mean squared error criterion the most recent and commonly applied estimators, such as the realized kernels of Barndorff-Nielsen, Hansen, Lunde & Shephard (2006), the two-scales realized variance of Zhang, Mykland & Aït-Sahalia (2005), the Hayashi & Yoshida (2005) covariance estimator, and the realized variance and covariance with the optimal sampling frequency chosen after Bandi & Russell (2005a) and Bandi & Russell (2005b).

The power of our methodology stems from the fact that instead of trying to correct the realized quantities for the noise, we identify both the true underlying integrated moments and the moments of the noise, which are also estimated within our framework. Apart from being simple to implement, an important property of our estimators is that they are quite robust to misspecifications of the noise process.

JEL classification: $G10, F31, C32$

Keywords: High frequency data, Realized volatility and covariance, Market microstructure

1 Introduction

This paper presents a unified approach for estimating the covariance of a multivariate diffusion process in the presence of market microstructure noise. Recently, the literature on estimating the variance of irregularly observed high-frequency financial prices has experienced a substantial development in the direction of relaxing the usual i.i.d noise assumption and constructing consistent estimators for the variance of the underlying efficient price process. Leading examples in this aspect are the realized kernels of Barndorff-Nielsen et al. (2006) and the two-scales realized volatility by Zhang et al. (2005).

The progress in estimating the covariance between two randomly observed diffusion processes has been somewhat more cumbersome, due to the additional complication of non-synchronicity. Nevertheless, recent contributions such as Hayashi & Yoshida (2005) and Corsi & Audrino (2007) have introduced consistent and bias-free estimators for non-synchronously observed processes, when there are no market frictions. Based on these approaches, Griffin & Oomen (2006), Voev & Lunde (2007) and Zhang (2006), among others, consider the properties of such estimators in the presence of measurement noise and propose certain extensions in order to correct for the impact of the noise.

Still, there does not exist a unified methodology which can serve for estimating both the variance and covariance of high-frequency noisy prices, thus allowing for the estimation of the whole variance-covariance matrix, which accounts for a large range of possible noise specifications and non-synchronicity. Our work is intended to fill this gap by providing precise and unbiased estimators, which are also easy to apply in practice. Furthermore, we obtain estimates on the dependence structure of the noise process, leading to a better understanding of the market microstructure frictions on the transaction level. The power of our methodology lies in the ability to separate the variation of the efficient price and noise processes, which jointly contribute to the variation of the observed noisy process. This identification is possible since the effect of the noise accumulates (up to a certain extent) by sampling more frequently, while the variation of the true process is constant. To give an intuition about our approach, consider the so-called volatility signature plots introduced by Andersen, Bollerslev, Diebold & Labys (1999) to study the properties of the realized variance estimator. In these plots there is a constant component (the integrated variance of the underlying process), while the noise variance accumulates linearly with the number of sampling points used to compute the realized variance. Since the number of sampling points is observable, the true integrated variance can be obtained as the constant on a projection of the realized volatility on the number of returns used to compute it. The paper is structured as follows: in Section 2 we setup the notation and the framework we are working under, Section 3 presents our estimation methodology, Section 4 contains the results of a simulation study in which we compare our approach to other existing approaches, and Section 5 concludes.

2 Theoretical Setup

Our basic assumption is that we have irregularly spaced, non-synchronous observations of an n-dimensional continuous time process $\mathbf{p}(t)$, $t \geq 0$, which is a noisy signal for an underlying process $\mathbf{p}^*(t)$:

$$\mathbf{p}(t) = \mathbf{p}^*(t) + \mathbf{u}(t),$$

where $\mathbf{u}(t)$ is the noise term. The elements of $\mathbf{p}$, $\mathbf{p}^*$ and $\mathbf{u}$ are denoted by p^k, p^{*k} and u^k, for $k = 1, \ldots, n$, respectively. The process $\mathbf{p}^*(t)$ satisfies the following

Assumption 1. *The process $\mathbf{p}^*(t)$ is a multivariate martingale process with stochastic volatility satisfying*

$$\mathbf{p}^*(t) = \int_0^t \mathbf{\Theta}(u) d\mathbf{W}(u)$$

where $\mathbf{\Theta}$ is the spot covolatility process and $\mathbf{W}$ is a vector standard Brownian motion of dimension q. All the elements of $\mathbf{\Theta}(t)\mathbf{\Theta}(t)'$ satisfy the Lipschitz condition.

Defining the spot covariance as $\mathbf{\Sigma}(t) = \mathbf{\Theta}(t)\mathbf{\Theta}(t)'$, the integrated covariation process of $\mathbf{p}^*$ is given by

$$\mathbf{IC}(t) = \int_0^t \mathbf{\Sigma}(u) du.$$

Our aim is to estimate the increment of integrated covariance

$$\mathbf{IC}(a, b) = \int_a^b \mathbf{\Sigma}(u) du = \mathbf{IC}(b) - \mathbf{IC}(a).$$

for some predetermined choice of (a, b), e.g., a trading day. Henceforth, we assume that the period of interest is a trading day and we will omit a and b in the notation. With respect to the market microstructure noise, we assume that it is independent (exogenous) of the underlying process p^*. Formally, we make the following assumption:

Assumption 2.

(*i*) $\mathbf{p}^*(s) \perp\!\!\!\perp \mathbf{u}(t)$, *for all* s *and* t*;*

(*ii*) $\mathrm{E}[\mathbf{u}(t)] = 0$ *for all* t*;*

Under this assumption the noise process can be serially correlated, but is exogenous to the true price process.

The i.i.d assumption has been relaxed in the univariate case by Barndorff-Nielsen et al. (2006) and Aït-Sahalia, Mykland & Zhang (2006), among others. In the univariate case, dependence in tick time is more intuitive and also easier to work with. In the multivariate case, the typical assumption has been i.i.d. noise (e.g. Griffin & Oomen (2006), Bandi & Russell (2005b)). Recently, Voev & Lunde (2007) showed that the i.i.d assumption does not hold and discussed the problems of defining dependence in tick-time in the multivariate case, therefore assuming serial cross-correlations in calendar time. In this paper we will follow this approach since it seems to be the most reasonable way to achieve an unified framework for modelling dependent noise processes in the multivariate framework with non-synchronicity. We can now complete Assumption 2 as follows

Assumption 2. *(continued)*

(*iii*) *The noise process* $\mathbf{u}$ *is covariance stationary with autocovariance function given by* $\mathbf{\Gamma}(q) = \mathrm{E}[\mathbf{u}(t)\mathbf{u}'(t-q)]$.

The (k,l)*-element of* $\mathbf{\Gamma}(q)$, $k,l = 1,\ldots,n$ *is denoted by* $\gamma_{k,l}(q)$.

Here we implicitly assume that time is measured in the smallest available time grid, e.g., in seconds. While we have until now considered the noise process as a continuous time process, in fact it is only present when transactions or quote updates take place, hence it makes more sense to treat it as occurring at the event times. Therefore we will use the notation $u^k(t_j^k) = u^k_{t_j^k}$ and $p^k(t_j^k) = p^k_{t_j^k}$, where t_j^k, $j = 1,\ldots,N^k$ denote the event (transaction, quotes, etc.) arrival times of asset $k = 1,\ldots,n$. Under Assumption 2, we have, e.g., that $\mathrm{E}\left[u^k_{t_j^k} u^l_{t^l_{j'}}\right] = \gamma_{k,l}(q)$, whenever $t_j^k - t^l_{j'} = q$. Here we note that there are alternative ways of specifying the function $\mathbf{\Gamma}(q)$, which preserve tick-time dependence for the univariate processes and defines cross-covariances for each pair (k,l) based on the pooled arrival process of both assets. The estimation approach presented in this paper is also applicable under alternative assumptions with slight modifications. In particular, if one is only interested in estimating variances, our model simplifies considerably under tick-time dependence. Cross-dependence in calender time can be motivated by staggered information assimilation in the prices of different assets or markets. While securities which are more closely followed by analysts and more frequently traded react faster to new information, slower trading assets take time to adjust to the news, causing lagged correlations across assets.

3 Estimation Procedures

If the process p^* were observed directly, a simple and asymptotically error-free estimator for $\mathbf{IC}(a, b)$ is the so-called realized covariance which is the sum of the squares of the increments of the process p^* at the highest available frequency over the interval (a, b). The properties of this estimator under such ideal conditions are derived in Barndorff-Nielsen & Shephard (2004). Two main issues arise for this estimator when used in practice. Firstly, when the separate univariate processes are not observed simultaneously, one has to resort to synchronization techniques in order to define joint observation times for the multivariate process. Such techniques lead to biases in the estimated covariances, which is known as the Epps effect (Epps (1979)). Secondly, the presence of noise leads to biases and inconsistency. The properties of the last-tick interpolation based realized covariances are studied by Zhang (2006), Griffin & Oomen (2006) and Martens (2004), among others. Stemming from these studies, different approaches are proposed to make the realized covariance robust to market microstructure noise such as calculation of optimal sampling frequency and lead-lag corrections. More recently, researchers have concentrated on developing in some cases rather sophisticated models which are specifically designed to estimate only the variance of a given asset (variance models) or a single covariance between two assets (covariance models). Concerning the variance models, recent advances include the two-scales realized variance by Zhang et al. (2005), the realized kernels of Barndorff-Nielsen et al. (2006), and the realized range-based variance which has newly been revived by Christensen & Podolskij (2007). With respect to covariance estimation Hayashi & Yoshida (2005) and Corsi & Audrino (2007) propose an estimator which does not require synchronization of observations and thus accounts for the Epps effect. Griffin & Oomen (2006) study the properties of this estimator under i.i.d. noise, while Voev & Lunde (2007) propose extensions to the Hayashi-Yoshida estimator to make it robust to market microstructure frictions of a general nature.

In our methodology, the variances and covariances are estimated separately, but within the same model framework. Advantages of our estimation procedure are its straightforward implementation and robustness to misspecifications of the noise process.

3.1 Variance Estimation

We focus first on estimating the integrated variance of a single asset and then we will turn to the covariance estimation. To separate the variance of the unobservable price process from the variance of the noise component we use the information contained in the volatility signature plot. Consider a given asset k with N^k observations (ticks, transactions, quote updates) within the period of interest. The volatility signature plot is the graphical representation of the realized variance against the sampling frequency at which it was computed, and was introduced by Andersen, Bollerslev, Diebold & Labys (2001). To this end the grid of observations $\{t_j^k\}_{j=1,\ldots,N^k}$ is subdivided into subgrids $\{t_{js+h}^k\}_{j=0,\ldots,\left\lfloor\frac{N^k-h}{s}\right\rfloor}$, where $s = 1,\ldots,S$ and $h = 1,\ldots,s$, which denotes the h-th subgrid for a sampling frequency of s ticks (e.g., with $s = 2$ we can have two subgrids, the first one comprising the ticks $\{t_1^k, t_3^k, t_5^k, \ldots\}$ and the second – the ticks $\{t_2^k, t_4^k, t_6^k, \ldots\}$). For each subgrid, we can define the corresponding observed and efficient s-tick returns as

$$
\begin{aligned}
r_{t_{js+h}}^k &= p_{t_{(j-1)s+h}}^k - p_{t_{js+h}}^k, \quad j = 1,\ldots,\left\lfloor\frac{N^k-h}{s}\right\rfloor \\
r_{t_{js+h}}^{*k} &= p_{t_{(j-1)s+h}}^{*k} - p_{t_{js+h}}^{*k}, \quad j = 1,\ldots,\left\lfloor\frac{N^k-h}{s}\right\rfloor,
\end{aligned}
$$

and the noise returns as

$$
e_{t_{js+h}}^k = u_{t_{(j-1)s+h}}^k - u_{t_{js+h}}^k, \quad j = 1,\ldots,\left\lfloor\frac{N^k-h}{s}\right\rfloor.
$$

Denote the number of returns for the h-th s-subgrid as $N_{h,s}^k = \left\lfloor\frac{N^k-h}{s}\right\rfloor - 1$. The realized variance of asset k based on this subgrid is defined explicitly as a function of the number of returns used as

$$
RV^k(N_{h,s}^k) = \sum_{j=1}^{N_{h,s}^k} \left(r_{t_{js+h}}^k\right)^2.
$$

For the variance estimation we will exploit the following relationship, which holds under Assumptions 1 and 2:

$$
\begin{aligned}
\mathrm{E}\left[RV^k(N^k_{h,s})\right] &= \mathrm{IV}^k + \sum_{j=1}^{N^k_{h,s}} \mathrm{Var}\left[e^k_{t_{js+h}}\right] \\
&= \mathrm{IV}^k + 2\sum_{q=1}^{\infty} N^k_{h,s}(q)\left(\gamma_{k,k}(0) - \gamma_{k,k}(q)\right) \\
&\approx \mathrm{IV}^k + 2N^k_{h,s}\gamma_{k,k}(0) - 2\sum_{q=1}^{Q} N^k_{h,s}(q)\gamma_{k,k}(q), \qquad (1)
\end{aligned}
$$

where IV^k is the integrated variance of the price process of asset k, i.e. element (k,k) of **IC** and $N^k_{h,s} = \sum_q N^k_{h,s}(q)$. Thereby, $N^k_{h,s}(q)$ counts the number of q-second returns of asset k for the (h,s)-subgrid given by

$$
N^k_{h,s}(q) = \sum_j \mathbb{1}_{\{t^k_{js+h} - t^k_{(j-1)s+h} = q\}}.
$$

Note that these counts need to be considered because we work with irregularly-spaced returns, which under the assumption of autocovariance function defined on the smallest regular time grid (each second), imply that each $\mathrm{Var}\left[e^k_{t_{js+h}}\right]$ consists of two elements, namely $\gamma_{k,k}(0)$ and the q-second autocovariance $\gamma_{k,k}(q)$. The approximation in equation (1) results from truncating the autocorrelation function at lag Q. This is reasonable, since for a covariance stationary process the autocovariance function tends to zero for large lags, so that Q has to be chosen appropriately. Also, from a practical point of view choosing Q too large leads to more estimation noise because for large Q's there are relatively few counts $N^k_{h,s}(Q)$. Furthermore, due to $N^k_{h,s} = \sum_q N^k_{h,s}(q)$, choosing Q too large leads to singularity of the regressor matrix.

Under the assumption of an i.i.d. noise process this yields the standard result (e.g., Barndorff-Nielsen & Shephard (2002)):

$$
\mathrm{E}\left[RV^k(N^k_{h,s})\right] = \mathrm{IV}^k + 2N^k_{h,s}\gamma_{k,k}(0),
$$

The only difference here is that we have to consider the q-th order autocorrelation of the noise process and we have to count the number of occurrences.

On the basis of this theoretical relationship and the above assumptions, we can easily derive the corresponding pooled OLS regression

$$
y_{h,s} = c + \beta' x_{h,s} + \varepsilon_{h,s}, \quad s = 1,\dots,S, \quad h = 1,\dots,s \qquad (2)
$$

where $y_{h,s} = RV^k(N^k_{h,s})$ and $x_{h,s}$ is the Q-dimensional vector given by $x_{h,s} = (N^k_{h,s}, N^k_{h,s}(1), \dots, N^k_{h,s}(Q))'$. In applications, Q has to be chosen appropriately, to reflect the degree of persistence of the noise process in the particular application.

In the above regression one simply regresses the realized variances on $N^k_{h,s}$ and the q-counts $N^k_{h,s}(q)$. The estimated constant $\hat{c}$ is an estimate of the integrated variance IV^k, while $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_Q$ are estimates of $2\gamma_{k,k}(0), -2\gamma_{k,k}(1), \ldots, -2\gamma_{k,k}(Q)$. Hence, as a byproduct of this estimation we can obtain the autocovariance function of the noise process, which can be identified under the assumption that the autocovariance $\gamma_{k,k}(Q)$ vanishes. For a particular application, one could choose Q in an iterative manner starting from a relatively small value which is increased in each step. The optimal value of Q is the smallest value at which a given criterion (e.g. the gradient of the estimates) does not change considerably anymore.
Furthermore, we can obtain the volatility signature plot as a plot of the pairs $(\iota' x_{h,s}, y_{h,s})$ for a given h or averaged across h, where ι is the vector of ones of dimension Q for $s = 1, \ldots, S$.

3.2 Covariance Estimation

Covariance estimation based on high-frequency data is inherently more challenging than variance estimation, since there is the additional complication of non-synchronicity. As mentioned already, non-synchronicity poses the problem of defining common event times for multiple assets. Typically, last-tick interpolation is employed, in which the last recorded price before a pre-defined observation time is taken as the observed price at that point of time. This leads to a bias towards zero in the estimated realized covariance as the sampling frequency increases. A solution to this problem is proposed by Hayashi & Yoshida (2005). Considering two assets k and l, the Hayashi-Yoshida (HY) estimator based on all observations is defined as

$$HY^{k,l} = \sum_{j=1}^{N^k} \sum_{j'=1}^{N^l} r^k_{t^k_j} r^l_{t^l_{j'}} \mathbb{1}_{\{(t^k_{j-1}, t^k_j] \cap (t^l_{j'-1}, t^l_{j'}]\}},$$

As can be seen from the definition, this estimator sums all cross products of overlapping returns of both assets. We can also base the estimation on the (h, s)- subgrid of asset k in conjunction with the (h', s')-subgrid of asset l, which we denote by

$$HY^{k,l}(h, h', s, s') = \sum_{j=1}^{N^k_{h,s}} \sum_{j'=1}^{N^l_{h',s'}} r^k_{t^k_{js+h}} r^l_{t^l_{j's'+h'}} \mathbb{1}_{\{(t^k_{(j-1)s+h}, t^k_{js+h}] \cap (t^l_{(j'-1)s'+h'}, t^l_{j's'+h'}]\}}. \tag{3}$$

In practice, it is convenient to implement this estimator by picking one of the assets, say k, and determining for each of its tick returns $r^k_{t^k_{js+h}}$, the corresponding return of the other asset which envelops it, i.e. starts before or at $t^k_{(j-1)s+h}$ and spans over at least to t^k_{js+h}. Of course, if one interchanges the assets, the estimator is numerically identical, but in terms of speed of execution, we recommend using the slower trading asset to determine the corresponding enveloping returns of the faster asset. In the following exposition we set the slower asset to be asset k. While the HY estimator is defined using all returns of both assets, effectively, there are at most $\min(N^k_{h,s}, N^l_{h',s'})$ different pairs of returns which contribute to the sum. This is so, because two or more neighboring returns of asset k may happen to be enveloped by the same return of asset l. Due to the summability of log returns, this is effectively only one return pair in the sum of the HY estimator. Thus, the amount of noise which accumulates in the sum is a function of such effective pairs, while some of the ticks t^k_{js+h} play no role and are hence irrelevant. In order to understand how this works we introduce the right- and left-continuous counting functions $N^k_{h,s}(t) = \sum_{j=1}^{N^k_{h,s}} \mathbb{1}_{\{t^k_{js+h} \le t\}}$ and $\breve{N}^k_{h,s}(t) = \sum_{j=1}^{N^k_{h,s}} \mathbb{1}_{\{t^k_{js+h} < t\}}$, $k = 1, \dots, n$, $s = 1, \dots, S$, and $h = 1, \dots, s$. The number of irrelevant ticks t^k_{js+h} can be computed as follows

$$NI^k(h,h',s,s') = \sum_{j=1}^{N^k_{h,s}} \mathbb{1}_{\left\{t^k_{js+h} \neq t^l_{N^l_{h',s'}\left(t^k_{js+h}\right)}\right\}} \mathbb{1}_{\left\{t^k_{(j+1)s+h} \le t^l_{N^l_{h',s'}\left(t^k_{js+h}\right)+1}\right\}} \mathbb{1}_{\left\{t^k_{(j-1)s+h} \ge t^l_{\breve{N}^l_{h',s'}\left(t^k_{js+h}\right)}\right\}} \tag{4}$$

Figure 1 illustrates graphically how such irrelevant ticks are obtained. The number of effective pairs is then given by

$$\tilde{N}^k(h,h',s,s') = N^k_{h,s} - NI^k(h,h',s,s').$$

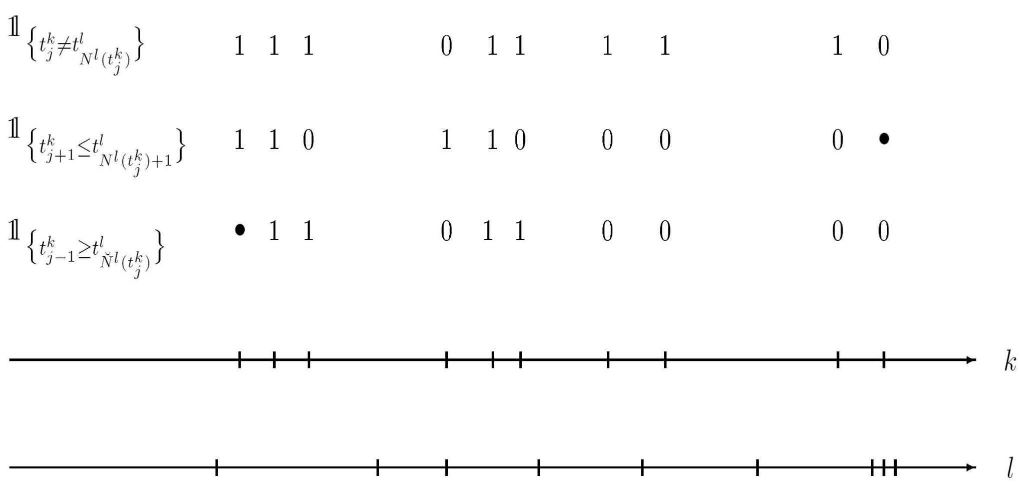

Figure 1: A graphical illustration for the identification of irrelevant ticks as described in equation (4). The indices h, h', s, s' have been suppressed.

Then the HY estimator can be rewritten as

$$
\begin{aligned}
HY^{k,l}(h,h',s,s') &= \sum_{j=1}^{N^k_{h,s}} r^k_{t^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(t^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(t^k_{js+h}\right)+1}} \\
&= \sum_{j=1}^{\tilde{N}^k(h,h',s,s')} r^k_{\tilde{t}^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}},
\end{aligned}
$$

where $r^l_{t^l_{j'}:t^l_{i'}}$ denotes the (possibly multiple-tick) return of asset l over the interval $(t^l_{j'}, t^l_{i'})$ and $\tilde{t}^k_{js+h}$ denote the relevant ticks of asset k in the (h,s)-subgrid, i.e., the set of all (h,s)-ticks minus the set of ticks fulfilling the condition in equation (4).
Each pair $r^k_{\tilde{t}^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}}$ can be decomposed as

$$
\begin{aligned}
r^k_{\tilde{t}^k_{js+h}} r^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} &= r^{*k}_{\tilde{t}^k_{js+h}} r^{*l}_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} \\
&\quad + e^k_{\tilde{t}^k_{js+h}} e^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}}. \qquad (5)
\end{aligned}
$$

The first product in the right-hand side of equation (5) contributes to the estimation of the integrated covariance, which we like to measure, while the second one is due to noise and we examine it further:

$$
\begin{aligned}
e^k_{\tilde{t}^k_{js+h}} e^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}:t^l_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} &= \left(u^k_{\tilde{t}^k_{js+h}} - u^k_{\tilde{t}^k_{(j-1)s+h}}\right)\left(u^l_{t_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}} - u^l_{t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}}\right) \\
&= \gamma_{k,l}\left(\overleftarrow{q}\right) + \gamma_{k,l}\left(\overrightarrow{q}\right) - \gamma_{k,l}\left(\overleftarrow{q}\right) - \gamma_{k,l}\left(\overrightarrow{q}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\overleftarrow{q} &= \tilde{t}^k_{(j-1)s+h} - t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)} \\
\overrightarrow{q} &= \tilde{t}^k_{js+h} - t_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1} \\
\overleftarrow{q} &= \tilde{t}^k_{js+h} - t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)} \\
\overrightarrow{q} &= \tilde{t}^k_{(j-1)s+h} - t_{\check{N}^l_{h',s'}\left(\tilde{t}^k_{js+h}\right)+1}
\end{aligned}
$$

are the time spans in seconds between the four returns endpoints and therefore the cross-correlation orders in the autocorrelation function $\gamma_{k,l}(q)$. Note that we always have $\overleftarrow{q} \geq 0$, $\overrightarrow{q} \leq 0$, $\overleftarrow{q} > 0$ and $\overrightarrow{q} < 0$. The number of $\overleftarrow{q}$ which are of a given length q is given by

$$
\overleftarrow{N}^k_{h,s,h',s'}(q) = \sum_{\overleftarrow{q}} \mathbb{1}_{\left\{\overleftarrow{q}=q\right\}} = \sum_j \mathbb{1}_{\left\{\tilde{t}^k_{(j-1)s+h} - t^l_{N^l_{h',s'}\left(\tilde{t}^k_{(j-1)s+h}\right)}=q\right\}}.
$$

Similarly, we can define $\overrightarrow{N}^k_{h,s,h',s'}(q)$, $\overleftarrow{N}^k_{h,s,h',s'}(q)$ and $\overrightarrow{N}^k_{h,s,h',s'}(q)$. Finally, the number of occurrences of $\gamma_{k,l}(q)$ in the expectation of the HY estimator will be

$$N^k_{h,s,h',s'}(q) = \overleftarrow{N}^k_{h,s,h',s'}(q) + \overrightarrow{N}^k_{h,s,h',s'}(q) - \overleftarrow{N}^k_{h,s,h',s'}(q) - \overrightarrow{N}^k_{h,s,h',s'}(q).$$

Then, under Assumptions 1 and 2 it holds that

$$\begin{aligned} \mathrm{E}\left[HY^{k,l}(h,h',s,s')\right] &= \mathrm{IC}^{k,l} + \sum_{q=-\infty}^{\infty} N^k_{h,s,h',s'}(q)\gamma_{k,l}(q) \\ &\approx \mathrm{IC}^{k,l} + \sum_{q=-Q}^{Q} N^k_{h,s,h',s'}(q)\gamma_{k,l}(q), \end{aligned}$$

where $\mathrm{IC}^{k,l}$ is the integrated covariance of the price processes of assets k and l, i.e. element (k,l) of $\mathbf{IC}$. The corresponding pooled OLS regression is

$$y_{h,s,h',s'} = c + \beta' x_{h,s,h',s'} + \varepsilon_{h,s,h',s'}, \qquad \begin{array}{ll} s = 1,\dots,S, & h = 1,\dots,s, \\ s' = 1,\dots,S', & h' = 1,\dots,s', \end{array} \tag{6}$$

where $y_{h,s,h',s'} = HY^{k,l}(h,h',s,s')$ and $x_{h,s,h',s'}$ is the $(2Q+1)$-dimensional vector given by $x_{h,s} = (N^k_{h,s,h',s'}(-Q), N^k_{h,s,h',s'}(-Q+1), \dots, N^k_{h,s,h',s'}(0), \dots, N^k_{h,s,h',s'}(Q-1), N^k_{h,s,h',s'}(Q))'$ and Q is chosen suitably.

4 Monte Carlo Study

In this section we detail the results of our Monte Carlo experiment designed to compare the bias and variance of a multitude of high-frequency volatility and covolatlity estimators for different trading scenarios.

4.1 Simulation Setup

We simulate two univariate price processes $p^{*k}(t)$ and $p^{*l}(t)$ as the following stochastic differential equations:

$$dp^{*k}(t) = \sigma_k(t)dW_k, \qquad dp^{*l}(t) = \sigma_l(t)dW_l, \tag{7}$$

where $\sigma_k(t)$ and $\sigma_l(t)$ follow GARCH diffusion processes given below and $\langle W_l, W_k\rangle_t = \rho$, i.e. the efficient price processes have stochastic volatility but constant correlation and hence the covariation process is also stochastic. While this setup can be extended by allowing for stochastic correlation, we find this unnecessary here.[1] The volatility processes are modelled as in Andersen & Bollerslev (1998) by

$$
\begin{aligned}
d\sigma_k^2(t) &= \lambda_k(\omega_k - \sigma_k^2(t))dt + \sqrt{2\lambda_k\theta_k}\sigma_k^2(t)dW_k^\sigma(t) \\
d\sigma_l^2(t) &= \lambda_l(\omega_l - \sigma_l^2(t))dt + \sqrt{2\lambda_l\theta_l}\sigma_l^2(t)dW_l^\sigma(t)
\end{aligned}
$$

where $\lambda_k > 0$, $\lambda_l > 0$, $\omega_k > 0$, $\omega_l > 0$, $0 > \theta_k > 1$, $0 > \theta_l > 1$ and the Brownian motions W_k^σ and W_l^σ are independent and also independent from W_k and W_l. Within this framework the integrated covariation matrix is given by

$$
\mathbf{IC} = \int_0^1 \begin{pmatrix} \sigma_k^2(t) & \bullet \\ \rho\sigma_k(t)\sigma_l(t) & \sigma_l^2(t) \end{pmatrix} dt
$$

The price and volatility processes are generated on a grid of one second for a total of 23400 seconds, corresponding to a typical NYSE trading day. The parameter values we use are as follows: $\lambda_k = 0.296$, $\lambda_l = 0.480$, $\omega_k = 0.636$, $\omega_l = 0.476$, $\theta_k = 0.035$, $\theta_l = 0.054$. These values have been obtained by Andersen & Bollerslev (1998) for the DM/USD and JPY/USD exchange rates.

The noise processes u_t^k and u_t^l are generated as a bivariate VAR(1) process on the same grid as the price processes:

$$
\begin{pmatrix} u_t^k \\ u_t^l \end{pmatrix} = \Phi \begin{pmatrix} u_{t-1}^k \\ u_{t-1}^l \end{pmatrix} + \begin{pmatrix} \varepsilon_t^k \\ \varepsilon_t^l \end{pmatrix},
$$

where the matrix $\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$ fulfils the conditions for stationarity of a VAR(1) process and

$$
\varepsilon_t \equiv \begin{pmatrix} \varepsilon_t^k \\ \varepsilon_t^l \end{pmatrix} \sim N(0, \Sigma_\varepsilon)
$$

is a bivariate white noise process. Obviously, the i.i.d. noise case is obtained for $\Phi = 0$, while the two noise processes are i.i.d. and uncorrelated across assets if $\Phi = 0$ and Σ_ε is diagonal.

After both the price and the noise processes are generated we obtain the noisy prices

$$
\mathbf{p}_t = \mathbf{p}_t^* + \mathbf{u}_t
$$

[1] Voev & Lunde (2007) find that stochastic vs. constant correlation, in the case with stochastic volatility does not influence the performance of the covariance estimators in their simulation study.

for each $t = 1, \ldots, 23400$ on the second-grid. To obtain different scenarios in terms of trading activity we generate random Poisson sampling times with constant intensities η_k and η_l for asset k and l, respectively.

In our simulation study, we keep the parameters pertaining to the volatility specification fixed, while we vary the parameters Φ, Σ_ε, η_k and η_l to reproduce various noise and trading intensity scenarios. The parameter constellations we consider are presented in Table 1. We consider three types of noise: i.i.d., dependent with low persistence, and dependent with high persistence. Each of these specifications is combined with large, moderate and low variances (diagonal elements of Σ_ε) of the white noise process ε_t. Furthermore, we generate observation times with very high, moderate and low intensities, to arrive finally at 27 scenarios.

Scenario	ϕ_{11}	ϕ_{12}	ϕ_{21}	ϕ_{22}	$\text{Var}(\varepsilon_k)$	$\text{Var}(\varepsilon_l)$	η_k	η_l
iid, low var., high int.	0	0	0	0	0.0001	0.0002	$\frac{1}{2}$	$\frac{1}{4}$
iid, low var., mod. int.	0	0	0	0	0.0001	0.0002	$\frac{1}{5}$	$\frac{1}{10}$
iid, low var., low int.	0	0	0	0	0.0001	0.0002	$\frac{1}{15}$	$\frac{1}{30}$
iid, mod. var., high int.	0	0	0	0	0.001	0.002	$\frac{1}{2}$	$\frac{1}{4}$
iid, mod. var., mod. int.	0	0	0	0	0.001	0.002	$\frac{1}{5}$	$\frac{1}{10}$
iid, mod. var., low int.	0	0	0	0	0.001	0.002	$\frac{1}{15}$	$\frac{1}{30}$
iid, high var., high int.	0	0	0	0	0.01	0.02	$\frac{1}{2}$	$\frac{1}{4}$
iid, high var., mod. int.	0	0	0	0	0.01	0.02	$\frac{1}{5}$	$\frac{1}{10}$
iid, high var., low int.	0	0	0	0	0.01	0.02	$\frac{1}{15}$	$\frac{1}{30}$
low pers., low var., high int.	0.4	0.1	0.2	0.5	0.0001	0.0002	$\frac{1}{2}$	$\frac{1}{4}$
low pers., low var., mod. int.	0.4	0.1	0.2	0.5	0.0001	0.0002	$\frac{1}{5}$	$\frac{1}{10}$
low pers., low var., low int.	0.4	0.1	0.2	0.5	0.0001	0.0002	$\frac{1}{15}$	$\frac{1}{30}$
low pers., mod. var., high int.	0.4	0.1	0.2	0.5	0.001	0.002	$\frac{1}{2}$	$\frac{1}{4}$
low pers., mod. var., mod. int.	0.4	0.1	0.2	0.5	0.001	0.002	$\frac{1}{5}$	$\frac{1}{10}$
low pers., mod. var., low int.	0.4	0.1	0.2	0.5	0.001	0.002	$\frac{1}{15}$	$\frac{1}{30}$
low pers., high var., high int.	0.4	0.1	0.2	0.5	0.01	0.02	$\frac{1}{2}$	$\frac{1}{4}$
low pers., high var., mod. int.	0.4	0.1	0.2	0.5	0.01	0.02	$\frac{1}{5}$	$\frac{1}{10}$
low pers., high var., low int.	0.4	0.1	0.2	0.5	0.01	0.02	$\frac{1}{15}$	$\frac{1}{30}$
high pers., low var., high int.	0.85	0.15	-0.1	0.85	0.0001	0.0002	$\frac{1}{2}$	$\frac{1}{4}$
high pers., low var., mod. int.	0.85	0.15	-0.1	0.85	0.0001	0.0002	$\frac{1}{5}$	$\frac{1}{10}$
high pers., low var., low int.	0.85	0.15	-0.1	0.85	0.0001	0.0002	$\frac{1}{15}$	$\frac{1}{30}$
high pers., mod. var., high int.	0.85	0.15	-0.1	0.85	0.001	0.002	$\frac{1}{2}$	$\frac{1}{4}$
high pers., mod. var., mod. int.	0.85	0.15	-0.1	0.85	0.001	0.002	$\frac{1}{5}$	$\frac{1}{10}$
high pers., mod. var., low int.	0.85	0.15	-0.1	0.85	0.001	0.002	$\frac{1}{15}$	$\frac{1}{30}$
high pers., high var., high int.	0.85	0.15	-0.1	0.85	0.01	0.02	$\frac{1}{2}$	$\frac{1}{4}$
high pers., high var., mod. int.	0.85	0.15	-0.1	0.85	0.01	0.02	$\frac{1}{5}$	$\frac{1}{10}$
high pers., high var., low int.	0.85	0.15	-0.1	0.85	0.01	0.02	$\frac{1}{15}$	$\frac{1}{30}$

Table 1: Monte Carlo Simulation Scenarios. We use the following abbreviations: "var." stands for variance, "mod." stands for moderate, "pers." stands for persistence, "int." stands for intensity. The correlation between ε_k and ε_l is set in all scenarios equal to $\text{Corr}(\varepsilon_k, \varepsilon_l) = -0.1$.

4.2 Estimators

We include a wide set of estimators to compete against our proposed methodology. For the univariate case we consider the standard realized volatility at different sampling frequencies, including the optimal sampling frequency derived in Bandi & Russell (2005a), realized volatility with lag correction, the realized kernels of Barndorff-Nielsen et al. (2006) and the two-scales estimator of Zhang et al. (2005). For the estimation of the integrated covariance we consider the realized covariance computed at different sampling frequencies, including the optimal sampling frequency derived in Bandi &

Russell (2005b), realized covariance with lead/lag correction, and the HY estimator and its subsampled version. Our estimators are the estimated constants in the OLS regressions in equations (2) and (6) for the integrated variance and covariance, respectively. In our approach there are two parameters that need to be chosen: Q – the number of lags for the (cross) autocovariance function of the noise processes, and S – the number of subsamples. We discuss on the choice of these parameters after we setup the notation for the estimators.

The standard realized variance is denoted by $RV(\delta)$, where δ is sampling frequency in seconds, while by $RV_L(\delta)$ we denote the realized variance with a lag correction of L lags. For the realized kernels we use the notation $K^{TH2}(\delta)$ for the modified Tukey-Hanning kernel as described in Barndorff-Nielsen et al. (2006). The two scales realized variance of Zhang et al. (2005) is a combination of an averaged subsampled realized variance at moderate frequencies combined with a very high frequency realized variance correction term and is denoted by $TSRV$.

The usual last-tick interpolation realized covariance is denoted by $RC(\delta)$, while its biased corrected version, with L^+ leads and L^- lags, is denoted by $RC_{L^+,L^-}(\delta)$. The Hayashi-Yoshida estimator is denoted as above by HY and its subsampled version based on S subsamples by $HY(S)$. Finally, we denote our estimators by $NV(S,Q)$ in the univariate case, and $NV(S,S',Q^+,Q^-)$ in the multivariate case. In our Monte Carlo study, we set $S'=1$ for simplicity, i.e., we only consider subsamples of the first asset. The estimators could be improved by subsampling the second asset as well.

For many of the estimators listed above, in order to determine optimal sampling frequencies or number of subgrids, one needs to determine in a first step the second moments of the noise process, as well as the integrated quarticity of the efficient price process, which we denote by IQ^k for asset k. While there are different estimators for these quantities, we adopt a simple approach by following Barndorff-Nielsen et al. (2006). Thus, the noise moments $\gamma_{k,k}(0)$, $\gamma_{l,l}(0)$ are obtained by averaging over subsampled realized variances computed at 60 seconds and dividing by twice the number of returns, while the integrated quarticity is obtained as the squared of the average over realized variance computed at 20 minutes. While there are currently better estimators for these quantities, this methodology is robust to a fairly large range of noise specifications and delivers reasonable estimates. In the multivariate case, we need to estimate $\gamma_{k,l}(0)$ and a quantity which corresponds to the integrated quarticity, which we denote by $\mathrm{IQ}^{k,l}$ and is given by

$$\mathrm{IQ}^{k,l} = \int_0^1 \sigma_{k,k}(t)\sigma_{l,l}(t) + \sigma_{k,l}^2(t)dt,$$

where $\sigma_{k,l}(t)$ is the (k,l)-element of $\mathbf{\Sigma}(t)$. To estimate these quantities we rely on the approach proposed by Bandi & Russell (2005b). Putting everything together, the optimal sampling frequency is given as $\delta^* = \lceil \frac{23400}{N^*} \rceil$, where N^* is determined by

$$N^* = \begin{cases} \left(\frac{\mathrm{IQ}^k}{\gamma_{k,k}^2(0)} \right)^{\frac{1}{3}}, & \text{in the univariate case} \\ \left(\frac{\mathrm{IQ}^{k,l}}{2\gamma_{k,l}^2(0)} \right)^{\frac{1}{3}}, & \text{in the multivariate case.} \end{cases} \tag{8}$$

For the optimal number of subgrids, we rely on results derived in Zhang et al. (2005) in the univariate case and Voev & Lunde (2007) in the multivariate case. Thus we determine

$$S^* = c^{1/3} N^{2/3}, \tag{9}$$

where N is the total number of observations of the asset under consideration in the univariate case, while in the multivariate case, N is the number of observations of the slower asset and the optimal constant c is given by

$$c = \begin{cases} \frac{12\gamma_{k,k}^2(0)}{\mathrm{IQ}^k}, & \text{in the univariate case} \\ \frac{12\left(\gamma_{k,k}^2(0)\gamma_{l,l}^2(0) + \gamma_{k,l}^2(0)\right)}{\mathrm{IQ}^{k,l}}, & \text{in the multivariate case.} \end{cases}$$

4.3 Simulation Results

The results from the Monte Carlo study are collected in the Appendix. The main message is that our estimators clearly dominate all other considered estimators both in the univariate, as well as in the multivariate case! Our main competitors are as expected the realized kernel and the TSRV in the univariate case, and the Bandi & Russell (2005b) realized covariance as well as the HY-type estimators (for very particular cases) in the multivariate case.

The considered realized kernel is the only other estimator, apart from our estimator, that delivers unbiased estimates across the range of Monte Carlo scenarios. It is, however, clearly outperformed in the iid. case by the TSRV, while our estimator is not. In order to check whether it could be that the inputs required for the construction of the realized kernels and the TSRV impairs their performance, we computed their infeasible versions by setting the unknown quantities (e.g., the integrated quarticity or noise variance) to their true values. Overall, this did not qualitatively influence the results, implying that the estimators have been constructed reasonably.

In the covariance estimation, the subsampled HY estimator is of equal quality to our estimator only in the very special case of iid. noise with low variance and moderate or low trading intensity. In all other cases, it is severely biased and hence not competitive. The Bandi & Russell (2005b) estimator with a first-order lead/lag correction is performing quite well and is after our estimator the second-best alternative.

A very nice feature of our approach is that the proposed estimator is very robust and not too sensitive to the choice of the number of subsamples S. What is important, however, is that Q is chosen reasonably, which on the one hand means that it should not be too low (omitted variable problem) in the case of highly persistent noise, and on the other hand not too close to $S(S+1)/2$ (the number of observations in the pooled OLS regression) to assure that the X matrix is not close to being singular.

In table 2 we report some summary statistics of the ranks of all considered estimators, according to their root mean squared error, across simulation scenarios. The models $NV(S^{*}, 10)$, $NV(S^{*}, 20)$ (in the univariate case) and $NV(S^{*}, 1, 10, 10)$, $NV(S^{*}, 1, 20, 20)$ (in the multivariate case) are not considered in the ranking, because the S, Q-combinations produced in several scenarios (low intensity specifications) a near-singular X matrix. These S, Q-combinations should be avoided and can be identified by a nearly perfect fit in the OLS regression (R-squared very close to one and sum of squared residuals almost identical to zero).

	IV^1				IV^2			
Model	Mean	Median	Min	Max	Mean	Median	Min	Max
$RV(5)$	19.9	20	18	20	20.0	20	19	20
$RV(300)$	13.4	13	5	18	15.1	18	9	18
$RV(900)$	12.7	12	8	17	13.6	12	9	17
$RV(1800)$	13.3	13	7	17	13.4	14	8	16
$RV_1(5)$	17.6	19	5	19	18.9	19	18	19
$RV_1(300)$	11.1	11	6	14	11.9	13	7	15
$RV_1(900)$	14.1	16	8	19	13.6	15	9	17
$RV_1(1800)$	15.8	18	7	20	14.6	17	6	20
$RV(\delta^*)$	11.7	11	6	16	12.6	13	7	16
$RV_1(\delta^*)$	12.1	13	6	15	12.1	12	7	14
$RV_2(\delta^*)$	14.2	15	8	16	14.0	15	8	16
$K^{TH2}(60)$	8.4	9	5	12	8.4	9	5	11
$TSRV$	8.8	7	2	18	8.1	8	2	18
$NV(S^*,0)$	8.0	8	1	18	7.6	8	1	17
$NV(2S^*,0)$	5.5	3	1	16	4.3	3	1	15
$NV(3S^*,0)$	5.1	4	1	14	3.8	4	1	10
$NV(S^*,10)$	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.
$NV(2S^*,10)$	3.6	4	1	6	3.5	4	1	6
$NV(3S^*,10)$	5.0	5	1	8	4.8	5	1	8
$NV(S^*,20)$	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.	n.r.
$NV(2S^*,20)$	4.1	4	1	8	4.3	4	2	7
$NV(3S^*,20)$	5.7	6	1	9	5.6	6	1	8
$NV(BIC)$	2.9	3	1	7	2.8	3	1	8
$NV(BIC^*)$	3.0	3	1	8	2.5	2	1	8

Table 2: Mean, median, maximum and minimum of the root mean squared error rankings across simulation scenarios. "n.r." stands for not ranked. $NV(BIC)$ stands for the model with the smallest Bayesian Information Criterion across all NV estimators for each scenario. $NV(BIC^*)$ stands for the model with the smallest modified Bayesian Information Criterion (equation (10)) across all NV estimators for each scenario.

	IC			
Model	Mean	Median	Min	Max
$RC(5)$	16.4	18	1	20
$RC(300)$	11.8	13	8	16
$RC(900)$	12.0	12	7	16
$RC(1800)$	13.2	14	7	18
$RC_{1,1}(5)$	16.4	18	6	20
$RC_{1,1}(300)$	12.9	13	8	17
$RC_{1,1}(900)$	14.7	17	8	19
$RC_{1,1}(1800)$	15.8	18	6	20
$RC(\delta^*)$	13.2	14	4	18
$RC_{1,1}(\delta^*)$	10.0	10	5	17
$RC_{2,2}(\delta^*)$	11.7	11	8	16
HY	15.8	18	2	20
$HY(S^*)$	10.9	13	1	17
$NV(S^*, 1, 0, 0)$	5.1	3	1	15
$NV(2S^*, 1, 0, 0)$	3.3	2	1	9
$NV(3S^*, 1, 0, 0)$	3.8	4	1	6
$NV(S^*, 1, 10, 10)$	n.r.	n.r.	n.r.	n.r.
$NV(2S^*, 1, 10, 10)$	4.1	3	1	11
$NV(3S^*, 1, 10, 10)$	5.0	5	1	10
$NV(S^*, 1, 20, 20)$	n.r.	n.r.	n.r.	n.r.
$NV(2S^*, 1, 20, 20)$	6.1	5	2	15
$NV(3S^*, 1, 20, 20)$	8.0	7	2	20
$NV(BIC)$	3.9	3	1	15
$NV(BIC^*)$	2.9	3	1	6

Table 2 ***(cont'd)*****:** Mean, median, maximum and minimum of the root mean squared error rankings across simulation scenarios. "n.r." stands for not ranked. $NV(BIC)$ stands for the model with the smallest Bayesian Information Criterion across all NV estimators for each scenario. $NV(BIC^*)$ stands for the model with the smallest modified Bayesian Information Criterion (equation (10)) across all NV estimators for each scenario.

Table 2 contains the mean, median, minimum and maximum rank (according to the smallest RMSE) for each estimator across Monte Carlo scenarios. We propose two criteria to choose the proper combination of the parameters S and Q both based on the best fit of the regression, given by

$$
\begin{aligned}
BIC &= \ln\left(\frac{1}{n}\sum_{h,s}\hat{\varepsilon}^2_{h,s}\right) + \frac{p\ln n}{n}, \quad \text{with } n = \frac{S(S+1)}{S} \\
BIC^* &= \ln\left(\frac{1}{n}\sum_{h,s}\frac{\hat{\varepsilon}^2_{h,s}}{s}\right) + \frac{p\ln n}{n}, \quad \text{with } n = S,
\end{aligned}
\tag{10}
$$

where $\hat{\varepsilon}_{h,s}$ is the pooled OLS regression residual and p is the number of parameters in the regression. The first criterion is the usual Bayesian Information Criterion (BIC) for the pooled regression, while the second one is a modified BIC for a regression over $s = 1, \ldots, S$, where the squared residual for each s is the mean squared residual of the s-block. The modified BIC is motivated by the fact that the number of elements in the s-block, which equally contribute to the estimation, is linearly increasing with s. It accounts for the fact that with s getting large a single (h, s)-observation is becoming more noisy and therefore should be counted with an accordingly smaller weight.

An alternative model selection strategy is based on a procedure, where one starts from a high value of Q which is sequentially reduced. Within this procedure, it can be detected whether the estimates increase or decrease significantly as Q becomes smaller. If this is the case, it is an indication for the presence of persistence in the noise processes and consequently Q should be chosen preferably a bit too high rather than too low. Whenever Q is chosen to be large, it is beneficial to choose S large as well, since as mentioned above and explicit in the simulation results, one cannot choose S too bad by choosing it too large, which also alleviates the discussed near-singularity problem.

From Table 2 it is clear that our models are dominating and that the proposed selection criteria, although not perfect are doing a very good job. It is worth noting that there is at least one of our estimators that outperforms all others in *each* Monte Carlo scenario in the univariate case! In the multivariate case there are two exceptions: the $HY(S^*)$ ranks first in the iid., low variance, moderate intensity scenario, while the $RC(5)$ ranks first in the low persistence, low variance and high intensity scenario. The last case is a coincidence, in which the noise induced bias cancels exactly against the bias caused by the Epps effect.

5 Conclusion

The paper introduces a unified framework for the estimation of integrated second moments of irregularly observed asset prices contaminated by market microstructure noise. The estimation is performed under fairly weak assumptions on the dependence structure of the noise processes in a simple OLS regression framework. This approach allows for a robust estimation of the whole covariance matrix of asset returns in applications with large number of assets. Moreover, we can identify the dependence structure of the noise process, which sheds light on the properties of the market microstructure.

We derive the OLS regressions theoretically for the univariate and multivariate case and perform an extensive Monte Carlo study to compare the performance of our estimators against the most recent and commonly used approaches in the extant literature. The results are unequivocal: our estimators clearly dominate the other approaches across a comprehensive range of trading scenarios.

Promising directions for further research are on the one hand a more profound analysis of a model selection criterion based on the statistical properties of our estimators, relaxing the assumption of noise exogeneity, and on the other hand an empirical application with a large number of assets, e.g., in the field of asset pricing or risk management.

Appendix

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0002	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	1.5339 (0.0337)	1.9271 (0.0520)						
$RV(300)$	0.6311 (0.1025)	0.5382 (0.0905)						
$RV(900)$	0.6040 (0.1652)	0.5008 (0.1370)						
$RV(1800)$	0.5841 (0.2376)	0.4792 (0.1921)						
$RV_1(5)$	0.6512 (0.0372)	0.8554 (0.0453)						
$RV_1(300)$	0.6119 (0.1736)	0.5069 (0.1421)						
$RV_1(900)$	0.6021 (0.2902)	0.4917 (0.2374)						
$RV_1(1800)$	0.5777 (0.3977)	0.4924 (0.3377)						
$RV(\delta^*)$	0.6248 (0.1079)	0.5275 (0.0931)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_1(\delta^*)$	0.5978 (0.1725)	0.4953 (0.1479)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_2(\delta^*)$	0.6036 (0.2093)	0.5014 (0.1867)	0.0009 (0.0000)			0.0009 (0.0000)		
$K^{TH2}(60)$	0.6133 (0.1283)	0.5061 (0.1093)	0.0009 (0.0000)			0.0009 (0.0000)		
$TSRV$	0.6222 (0.0300)	0.5129 (0.0314)	0.0009 (0.0000)			0.0009 (0.0000)		
$NV(S^*,0)$	0.6240 (0.0279)	0.5144 (0.0298)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(2S^*,0)$	0.6239 (0.0361)	0.5138 (0.0365)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(3S^*,0)$	0.6236 (0.0424)	0.5132 (0.0423)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(S^*,10)$	0.6232 (0.0410)	0.5131 (0.0438)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0002 (0.0000)	0.0000 (0.0006)	-0.0001 (0.0019)
$NV(2S^*,10)$	0.6238 (0.0480)	0.5131 (0.0468)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0002 (0.0000)	0.0000 (0.0007)	-0.0001 (0.0020)
$NV(3S^*,10)$	0.6232 (0.0536)	0.5124 (0.0521)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0000)	0.0000 (0.0007)	-0.0001 (0.0020)
$NV(S^*,20)$	0.6236 (0.0719)	0.5162 (0.0848)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0001)	0.0000 (0.0011)	-0.0001 (0.0034)
$NV(2S^*,20)$	0.6239 (0.0539)	0.5131 (0.0516)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0000)	0.0000 (0.0011)	-0.0001 (0.0034)
$NV(3S^*,20)$	0.6229 (0.0589)	0.5122 (0.0565)	0.0001 (0.0000)	0.0000 (0.0000)	0.0000 (0.0001)	0.0002 (0.0000)	0.0000 (0.0011)	-0.0001 (0.0036)

Table 3: Monte Carlo simulation results for scenario: **iid, low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.0886 (0.0269)					
$RC(300)$	0.1944 (0.0737)					
$RC(900)$	0.1853 (0.1215)					
$RC(1800)$	0.1820 (0.1657)					
$RC_{1,1}(5)$	0.1750 (0.0287)					
$RC_{1,1}(300)$	0.1942 (0.1225)					
$RC_{1,1}(900)$	0.1925 (0.2017)					
$RC_{1,1}(1800)$	0.1933 (0.2865)					
$RC(\delta^*)$	0.1473 (0.0370)			0.0000 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1958 (0.0353)			0.0000 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1968 (0.0415)			0.0000 (0.0000)		
HY	0.1177 (0.0303)					
$HY(S^*)$	0.1750 (0.0191)					
$NV(S^*, 1, 0, 0)$	0.1990 (0.0244)			-0.0000 (0.0000)		
$NV(2S^*, 1, 0, 0)$	0.1987 (0.0299)			-0.0000 (0.0000)		
$NV(3S^*, 1, 0, 0)$	0.1985 (0.0344)			-0.0000 (0.0000)		
$NV(S^*, 1, 10, 10)$	0.1996 (0.0365)	-0.0000 (0.0020)	0.0000 (0.0019)	-0.0000 (0.0001)	-0.0000 (0.0020)	-0.0000 (0.0021)
$NV(2S^*, 1, 10, 10)$	0.1985 (0.0385)	0.0000 (0.0016)	0.0001 (0.0016)	-0.0000 (0.0001)	-0.0000 (0.0016)	-0.0000 (0.0016)
$NV(3S^*, 1, 10, 10)$	0.1982 (0.0425)	0.0000 (0.0015)	0.0000 (0.0014)	-0.0000 (0.0000)	-0.0000 (0.0015)	-0.0000 (0.0015)
$NV(S^*, 1, 20, 20)$	0.1991 (0.1122)	0.0001 (0.0056)	0.0001 (0.0061)	-0.0000 (0.0001)	-0.0001 (0.0057)	-0.0001 (0.0064)
$NV(2S^*, 1, 20, 20)$	0.1982 (0.0424)	0.0001 (0.0033)	0.0002 (0.0034)	-0.0000 (0.0001)	-0.0002 (0.0033)	-0.0001 (0.0034)
$NV(3S^*, 1, 20, 20)$	0.1980 (0.0463)	0.0001 (0.0029)	0.0001 (0.0030)	-0.0000 (0.0001)	-0.0001 (0.0029)	-0.0001 (0.0029)

Table 3 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0002	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	1.2608 (0.0349)	1.2613 (0.0473)						
$RV(300)$	0.6296 (0.1032)	0.5400 (0.0896)						
$RV(900)$	0.6042 (0.1643)	0.5011 (0.1393)						
$RV(1800)$	0.5837 (0.2350)	0.4804 (0.1928)						
$RV_1(5)$	0.8253 (0.0364)	0.9587 (0.0454)						
$RV_1(300)$	0.6117 (0.1718)	0.5076 (0.1433)						
$RV_1(900)$	0.6015 (0.2890)	0.4922 (0.2384)						
$RV_1(1800)$	0.5788 (0.3970)	0.4921 (0.3370)						
$RV(\delta^*)$	0.6276 (0.1054)	0.5298 (0.0968)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_1(\delta^*)$	0.6025 (0.1739)	0.4926 (0.1527)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_2(\delta^*)$	0.6042 (0.2106)	0.5002 (0.1872)	0.0009 (0.0000)			0.0009 (0.0000)		
$K^{TH2}(60)$	0.6125 (0.1347)	0.5054 (0.1146)	0.0009 (0.0000)			0.0009 (0.0000)		
$TSRV$	0.6220 (0.0379)	0.5115 (0.0402)	0.0009 (0.0000)			0.0009 (0.0000)		
$NV(S^*, 0)$	0.6243 (0.0362)	0.5148 (0.0391)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(2S^*, 0)$	0.6232 (0.0453)	0.5134 (0.0466)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(3S^*, 0)$	0.6220 (0.0521)	0.5127 (0.0534)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(S^*, 10)$	0.6234 (0.0537)	0.5141 (0.0761)	0.0001 (0.0000)	-0.0000 (0.0011)	0.0001 (0.0066)	0.0002 (0.0001)	-0.0014 (0.0971)	-0.0031 (0.1609)
$NV(2S^*, 10)$	0.6219 (0.0590)	0.5120 (0.0607)	0.0001 (0.0000)	-0.0000 (0.0012)	-0.0000 (0.0069)	0.0002 (0.0001)	-0.0008 (0.0831)	-0.0005 (0.1485)
$NV(3S^*, 10)$	0.6205 (0.0647)	0.5116 (0.0666)	0.0001 (0.0000)	-0.0001 (0.0014)	0.0001 (0.0070)	0.0002 (0.0001)	-0.0005 (0.0799)	-0.0003 (0.1605)
$NV(S^*, 20)$	0.6198 (0.0930)	0.5141 (0.6518)	0.0001 (0.0001)	0.0000 (0.0032)	0.0003 (0.0174)	0.0002 (0.0009)	0.0034 (0.8386)	0.0132 (1.2880)
$NV(2S^*, 20)$	0.6212 (0.0637)	0.5119 (0.0676)	0.0001 (0.0000)	0.0000 (0.0029)	0.0000 (0.0158)	0.0002 (0.0001)	0.0147 (0.5689)	-0.0066 (0.4001)
$NV(3S^*, 20)$	0.6198 (0.0689)	0.5115 (0.0722)	0.0001 (0.0001)	0.0000 (0.0029)	0.0002 (0.0146)	0.0002 (0.0001)	0.0121 (0.5493)	-0.0088 (0.4023)

Table 4: Monte Carlo simulation results for scenario: **iid, low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.0496 (0.0192)					
$RC(300)$	0.1912 (0.0745)					
$RC(900)$	0.1858 (0.1218)					
$RC(1800)$	0.1824 (0.1659)					
$RC_{1,1}(5)$	0.1216 (0.0261)					
$RC_{1,1}(300)$	0.1949 (0.1229)					
$RC_{1,1}(900)$	0.1913 (0.2006)					
$RC_{1,1}(1800)$	0.1926 (0.2853)					
$RC(\delta^*)$	0.1435 (0.0359)			0.0000 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1930 (0.0429)			0.0000 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1944 (0.0522)			0.0000 (0.0000)		
HY	0.1864 (0.0303)					
$HY(S^*)$	0.1931 (0.0272)					
$NV(S^*, 1, 0, 0)$	0.1987 (0.0337)			-0.0000 (0.0000)		
$NV(2S^*, 1, 0, 0)$	0.1983 (0.0378)			-0.0000 (0.0001)		
$NV(3S^*, 1, 0, 0)$	0.1982 (0.0427)			-0.0000 (0.0001)		
$NV(S^*, 1, 10, 10)$	0.1982 (0.0523)	0.0001 (0.0026)	-0.0000 (0.0013)	-0.0000 (0.0006)	-0.0000 (0.0033)	0.0001 (0.0017)
$NV(2S^*, 1, 10, 10)$	0.1980 (0.0460)	0.0000 (0.0014)	0.0000 (0.0013)	-0.0000 (0.0004)	-0.0000 (0.0014)	-0.0000 (0.0017)
$NV(3S^*, 1, 10, 10)$	0.1980 (0.0507)	-0.0000 (0.0010)	-0.0000 (0.0010)	-0.0000 (0.0003)	0.0000 (0.0010)	0.0000 (0.0011)
$NV(S^*, 1, 20, 20)$	0.1982 (0.0523)	0.0001 (0.0026)	-0.0000 (0.0013)	-0.0000 (0.0006)	-0.0000 (0.0033)	0.0001 (0.0017)
$NV(2S^*, 1, 20, 20)$	0.1977 (0.0533)	0.0002 (0.0071)	0.0001 (0.0068)	0.0000 (0.0007)	-0.0003 (0.0088)	-0.0001 (0.0079)
$NV(3S^*, 1, 20, 20)$	0.1979 (0.0564)	-0.0001 (0.0023)	-0.0001 (0.0022)	-0.0000 (0.0003)	0.0001 (0.0024)	0.0001 (0.0023)

Table 4 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6265	0.5148	0.0001	0.0000	0.0000	0.0002	0.0000	0.0000
$RV(5)$	0.9051	0.7967						
	(0.0399)	(0.0516)						
$RV(300)$	0.6294	0.5399						
	(0.1026)	(0.0904)						
$RV(900)$	0.6025	0.5016						
	(0.1676)	(0.1390)						
$RV(1800)$	0.5830	0.4793						
	(0.2389)	(0.1917)						
$RV_1(5)$	0.8213	0.7502						
	(0.0402)	(0.0516)						
$RV_1(300)$	0.6115	0.5074						
	(0.1733)	(0.1426)						
$RV_1(900)$	0.5990	0.4914						
	(0.2920)	(0.2371)						
$RV_1(1800)$	0.5783	0.4903						
	(0.3974)	(0.3356)						
$RV(\delta^*)$	0.6250	0.5295	0.0009			0.0008		
	(0.1032)	(0.0913)	(0.0001)			(0.0001)		
$RV_1(\delta^*)$	0.5979	0.4928	0.0009			0.0008		
	(0.1716)	(0.1496)	(0.0001)			(0.0001)		
$RV_2(\delta^*)$	0.6008	0.4980	0.0009			0.0008		
	(0.2134)	(0.1819)	(0.0001)			(0.0001)		
$K^{TH2}(60)$	0.6127	0.5069	0.0009			0.0008		
	(0.1452)	(0.1233)	(0.0001)			(0.0001)		
$TSRV$	0.6211	0.5080	0.0009			0.0008		
	(0.0516)	(0.0608)	(0.0001)			(0.0001)		
$NV(S^*, 0)$	0.6247	0.5140	0.0001			0.0002		
	(0.0495)	(0.0608)	(0.0000)			(0.0000)		
$NV(2S^*, 0)$	0.6233	0.5126	0.0001			0.0002		
	(0.0580)	(0.0656)	(0.0000)			(0.0000)		
$NV(3S^*, 0)$	0.6216	0.5125	0.0001			0.0002		
	(0.0657)	(0.0724)	(0.0000)			(0.0001)		
$NV(S^*, 10)$	0.6169	0.5125	0.0001	0.0007	0.0004	0.0002	-0.0003	0.0004
	(0.1398)	(0.0949)	(0.0002)	(0.0141)	(0.0110)	(0.0001)	(0.0275)	(0.0288)
$NV(2S^*, 10)$	0.6212	0.5113	0.0001	0.0005	-0.0004	0.0002	-0.0009	0.0007
	(0.0753)	(0.0805)	(0.0001)	(0.0095)	(0.0337)	(0.0001)	(0.0301)	(0.0220)
$NV(3S^*, 10)$	0.6192	0.5122	0.0001	0.0009	-0.0001	0.0002	0.0001	0.0003
	(0.0818)	(0.0861)	(0.0001)	(0.0088)	(0.0067)	(0.0001)	(0.0425)	(0.0269)
$NV(S^*, 20)$	0.6169	0.5125	0.0001	0.0007	0.0004	0.0002	-0.0003	0.0004
	(0.1398)	(0.0949)	(0.0002)	(0.0141)	(0.0110)	(0.0001)	(0.0275)	(0.0288)
$NV(2S^*, 20)$	0.6212	0.5113	0.0001	0.0005	-0.0004	0.0002	-0.0009	0.0007
	(0.0753)	(0.0805)	(0.0001)	(0.0095)	(0.0337)	(0.0001)	(0.0301)	(0.0220)
$NV(3S^*, 20)$	0.6192	0.5122	0.0001	0.0009	-0.0001	0.0002	0.0001	0.0003
	(0.0818)	(0.0861)	(0.0001)	(0.0088)	(0.0067)	(0.0001)	(0.0425)	(0.0269)

Table 5: Monte Carlo simulation results for scenario: **iid, low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.0195 (0.0128)					
$RC(300)$	0.1782 (0.0725)					
$RC(900)$	0.1807 (0.1218)					
$RC(1800)$	0.1793 (0.1655)					
$RC_{1,1}(5)$	0.0550 (0.0203)					
$RC_{1,1}(300)$	0.1943 (0.1213)					
$RC_{1,1}(900)$	0.1903 (0.2015)					
$RC_{1,1}(1800)$	0.1909 (0.2852)					
$RC(\delta^*)$	0.1271 (0.0444)			0.0001 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1844 (0.0595)			0.0001 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1913 (0.0731)			0.0001 (0.0000)		
HY	0.1971 (0.0416)					
$HY(S^*)$	0.1979 (0.0412)					
$NV(S^*, 1, 0, 0)$	0.1993 (0.0519)			-0.0000 (0.0004)		
$NV(2S^*, 1, 0, 0)$	0.1986 (0.0574)			-0.0000 (0.0005)		
$NV(3S^*, 1, 0, 0)$	0.1984 (0.0631)			-0.0000 (0.0006)		
$NV(S^*, 1, 10, 10)$	0.1995 (0.0516)		0.0001 (0.0077)	0.0000 (0.0016)	-0.0002 (0.0049)	
$NV(2S^*, 1, 10, 10)$	0.1976 (0.0753)	0.0049 (0.0769)	-0.0048 (0.0926)	-0.0001 (0.0265)	0.0024 (0.1034)	0.0037 (0.0675)
$NV(3S^*, 1, 10, 10)$	0.1979 (0.0706)	-0.0001 (0.0050)	0.0000 (0.0040)	-0.0001 (0.0021)	-0.0000 (0.0039)	0.0001 (0.0064)
$NV(S^*, 1, 20, 20)$	0.1995 (0.0516)		0.0001 (0.0077)	0.0000 (0.0016)	-0.0002 (0.0049)	
$NV(2S^*, 1, 20, 20)$	0.1976 (0.0753)	0.0049 (0.0769)	-0.0048 (0.0926)	-0.0001 (0.0265)	0.0024 (0.1034)	0.0037 (0.0675)
$NV(3S^*, 1, 20, 20)$	0.1944 (0.3177)	-0.0004 (0.0157)	0.0004 (0.0228)	0.0001 (0.0435)	0.0002 (0.0953)	-0.0006 (0.0364)

Table 5 ***(cont'd)*:** Monte Carlo simulation results for scenario: **iid, low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6265	0.5148	0.0010	0.0000	0.0000	0.0020	0.0000	0.0000
$RV(5)$	9.7120	14.6357						
	(0.2363)	(0.4211)						
$RV(300)$	0.7703	0.8149						
	(0.1250)	(0.1385)						
$RV(900)$	0.6478	0.5870						
	(0.1771)	(0.1605)						
$RV(1800)$	0.6052	0.5205						
	(0.2455)	(0.2073)						
$RV_1(5)$	0.8891	3.9165						
	(0.2008)	(0.3269)						
$RV_1(300)$	0.6109	0.5107						
	(0.1850)	(0.1693)						
$RV_1(900)$	0.6064	0.4970						
	(0.2966)	(0.2539)						
$RV_1(1800)$	0.5780	0.4945						
	(0.4025)	(0.3457)						
$RV(\delta^*)$	0.7000	0.6388	0.0018			0.0027		
	(0.1560)	(0.1683)	(0.0001)			(0.0001)		
$RV_1(\delta^*)$	0.6065	0.5030	0.0018			0.0027		
	(0.2160)	(0.2239)	(0.0001)			(0.0001)		
$RV_2(\delta^*)$	0.6127	0.5112	0.0018			0.0027		
	(0.2686)	(0.2711)	(0.0001)			(0.0001)		
$K^{TH2}(60)$	0.6106	0.5040	0.0018			0.0027		
	(0.1460)	(0.1422)	(0.0001)			(0.0001)		
$TSRV$	0.6197	0.5082	0.0018			0.0027		
	(0.0406)	(0.0486)	(0.0001)			(0.0001)		
$NV(S^*, 0)$	0.6232	0.5137	0.0010			0.0020		
	(0.0369)	(0.0446)	(0.0000)			(0.0000)		
$NV(2S^*, 0)$	0.6232	0.5130	0.0010			0.0020		
	(0.0440)	(0.0496)	(0.0000)			(0.0000)		
$NV(3S^*, 0)$	0.6224	0.5124	0.0010			0.0020		
	(0.0513)	(0.0575)	(0.0000)			(0.0000)		
$NV(S^*, 10)$	0.6239	0.5128	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0004
	(0.0502)	(0.0570)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0044)	(0.0129)
$NV(2S^*, 10)$	0.6233	0.5124	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0004
	(0.0552)	(0.0591)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0044)	(0.0128)
$NV(3S^*, 10)$	0.6219	0.5118	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0004
	(0.0620)	(0.0670)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0044)	(0.0127)
$NV(S^*, 20)$	0.6247	0.5153	0.0010	-0.0000	0.0000	0.0020	0.0000	-0.0002
	(0.0638)	(0.0666)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0065)	(0.0196)
$NV(2S^*, 20)$	0.6231	0.5126	0.0010	-0.0000	0.0000	0.0020	-0.0000	-0.0002
	(0.0606)	(0.0636)	(0.0000)	(0.0001)	(0.0003)	(0.0001)	(0.0064)	(0.0192)
$NV(3S^*, 20)$	0.6215	0.5118	0.0010	0.0000	0.0000	0.0020	-0.0000	-0.0001
	(0.0670)	(0.0712)	(0.0001)	(0.0001)	(0.0003)	(0.0001)	(0.0063)	(0.0186)

Table 6: Monte Carlo simulation results for scenario: **iid, mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0001	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	-0.1155 (0.2055)					
$RC(300)$	0.1913 (0.0946)					
$RC(900)$	0.1834 (0.1336)					
$RC(1800)$	0.1818 (0.1738)					
$RC_{1,1}(5)$	0.1456 (0.1789)					
$RC_{1,1}(300)$	0.1931 (0.1354)					
$RC_{1,1}(900)$	0.1937 (0.2098)					
$RC_{1,1}(1800)$	0.1945 (0.2931)					
$RC(\delta^*)$	0.1012 (0.1440)			-0.0000 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1861 (0.1265)			-0.0000 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1893 (0.1285)			-0.0000 (0.0000)		
HY	-0.6135 (0.2205)					
$HY(S^*)$	0.0347 (0.0459)					
$NV(S^*, 1, 0, 0)$	0.1991 (0.0346)			-0.0001 (0.0000)		
$NV(2S^*, 1, 0, 0)$	0.1986 (0.0382)			-0.0001 (0.0000)		
$NV(3S^*, 1, 0, 0)$	0.1984 (0.0436)			-0.0001 (0.0000)		
$NV(S^*, 1, 10, 10)$	0.1993 (0.0473)	0.0001 (0.0075)	0.0002 (0.0074)	-0.0001 (0.0002)	-0.0002 (0.0075)	-0.0001 (0.0076)
$NV(2S^*, 1, 10, 10)$	0.1983 (0.0463)	-0.0000 (0.0057)	0.0000 (0.0056)	-0.0001 (0.0001)	-0.0001 (0.0056)	0.0000 (0.0056)
$NV(3S^*, 1, 10, 10)$	0.1983 (0.0514)	-0.0001 (0.0048)	-0.0000 (0.0047)	-0.0001 (0.0001)	0.0000 (0.0048)	0.0001 (0.0047)
$NV(S^*, 1, 20, 20)$	0.1982 (0.0581)	0.0003 (0.0128)	0.0004 (0.0130)	-0.0001 (0.0002)	-0.0004 (0.0129)	-0.0003 (0.0132)
$NV(2S^*, 1, 20, 20)$	0.1979 (0.0498)	-0.0000 (0.0080)	0.0000 (0.0080)	-0.0001 (0.0001)	-0.0000 (0.0079)	0.0000 (0.0079)
$NV(3S^*, 1, 20, 20)$	0.1981 (0.0548)	-0.0001 (0.0057)	-0.0001 (0.0057)	-0.0001 (0.0001)	0.0001 (0.0057)	0.0001 (0.0056)

Table 6 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0010	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0020	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	6.9896	7.9679						
	(0.2161)	(0.3148)						
$RV(300)$	0.7645	0.8202						
	(0.1273)	(0.1370)						
$RV(900)$	0.6488	0.5894						
	(0.1730)	(0.1685)						
$RV(1800)$	0.6027	0.5259						
	(0.2392)	(0.2111)						
$RV_1(5)$	2.6229	4.9430						
	(0.1705)	(0.2644)						
$RV_1(300)$	0.6128	0.5114						
	(0.1827)	(0.1727)						
$RV_1(900)$	0.6040	0.5014						
	(0.2924)	(0.2531)						
$RV_1(1800)$	0.5803	0.4932						
	(0.4018)	(0.3454)						
$RV(\delta^*)$	0.7047	0.6342	0.0018			0.0027		
	(0.1521)	(0.1665)	(0.0001)			(0.0001)		
$RV_1(\delta^*)$	0.6064	0.5012	0.0018			0.0027		
	(0.2202)	(0.2187)	(0.0001)			(0.0001)		
$RV_2(\delta^*)$	0.6114	0.5080	0.0018			0.0027		
	(0.2672)	(0.2667)	(0.0001)			(0.0001)		
$K^{TH2}(60)$	0.6110	0.5062	0.0018			0.0027		
	(0.1523)	(0.1464)	(0.0001)			(0.0001)		
$TSRV$	0.6178	0.5053	0.0018			0.0027		
	(0.0507)	(0.0628)	(0.0001)			(0.0001)		
$NV(S^*, 0)$	0.6235	0.5142	0.0010			0.0020		
	(0.0472)	(0.0587)	(0.0000)			(0.0001)		
$NV(2S^*, 0)$	0.6219	0.5127	0.0010			0.0020		
	(0.0533)	(0.0622)	(0.0000)			(0.0001)		
$NV(3S^*, 0)$	0.6206	0.5121	0.0010			0.0020		
	(0.0615)	(0.0698)	(0.0000)			(0.0001)		
$NV(S^*, 10)$	0.6236	0.5131	0.0010	-0.0001	0.0007	0.0020	-0.0088	-0.0119
	(0.0665)	(0.0819)	(0.0001)	(0.0057)	(0.0306)	(0.0001)	(0.3839)	(0.7118)
$NV(2S^*, 10)$	0.6208	0.5118	0.0010	-0.0002	0.0007	0.0020	-0.0040	-0.0130
	(0.0662)	(0.0747)	(0.0001)	(0.0056)	(0.0292)	(0.0001)	(0.3645)	(0.7165)
$NV(3S^*, 10)$	0.6194	0.5113	0.0010	-0.0002	0.0006	0.0020	-0.0047	-0.0130
	(0.0739)	(0.0815)	(0.0001)	(0.0055)	(0.0282)	(0.0001)	(0.3603)	(0.7218)
$NV(S^*, 20)$	0.6211	0.5159	0.0010	-0.0003	0.0029	0.0020	0.0986	-0.0202
	(0.0851)	(0.1128)	(0.0001)	(0.0131)	(0.0654)	(0.0002)	(2.2447)	(1.7459)
$NV(2S^*, 20)$	0.6198	0.5119	0.0010	-0.0003	0.0021	0.0020	0.0715	-0.0335
	(0.0706)	(0.0800)	(0.0001)	(0.0122)	(0.0569)	(0.0001)	(2.1169)	(1.5789)
$NV(3S^*, 20)$	0.6188	0.5113	0.0010	-0.0001	0.0016	0.0020	0.0707	-0.0395
	(0.0781)	(0.0865)	(0.0001)	(0.0119)	(0.0531)	(0.0002)	(2.0738)	(1.6021)

Table 7: Monte Carlo simulation results for scenario: **iid, mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0001	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.0043 (0.1204)					
$RC(300)$	0.1901 (0.0980)					
$RC(900)$	0.1864 (0.1359)					
$RC(1800)$	0.1830 (0.1751)					
$RC_{1,1}(5)$	0.1075 (0.1315)					
$RC_{1,1}(300)$	0.1936 (0.1394)					
$RC_{1,1}(900)$	0.1925 (0.2066)					
$RC_{1,1}(1800)$	0.1930 (0.2907)					
$RC(\delta^*)$	0.1160 (0.1212)			0.0000 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1802 (0.1095)			0.0000 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1933 (0.1122)			0.0000 (0.0000)		
HY	0.0726 (0.1324)					
$HY(S^*)$	0.1593 (0.0507)					
$NV(S^*, 1, 0, 0)$	0.1986 (0.0458)			-0.0001 (0.0002)		
$NV(2S^*, 1, 0, 0)$	0.1986 (0.0485)			-0.0001 (0.0002)		
$NV(3S^*, 1, 0, 0)$	0.1984 (0.0546)			-0.0001 (0.0002)		
$NV(S^*, 1, 10, 10)$	0.1980 (0.0592)	-0.0001 (0.0051)	0.0001 (0.0041)	-0.0002 (0.0016)	0.0001 (0.0044)	0.0003 (0.0049)
$NV(2S^*, 1, 10, 10)$	0.1984 (0.0563)	-0.0000 (0.0022)	-0.0000 (0.0023)	-0.0001 (0.0005)	0.0001 (0.0021)	0.0001 (0.0026)
$NV(3S^*, 1, 10, 10)$	0.1982 (0.0618)	-0.0000 (0.0017)	0.0000 (0.0018)	-0.0001 (0.0004)	0.0000 (0.0017)	0.0000 (0.0019)
$NV(S^*, 1, 20, 20)$	0.1991 (0.0964)	-0.0003 (0.0329)	-0.0000 (0.0274)	0.0000 (0.0047)	-0.0004 (0.0354)	0.0003 (0.0362)
$NV(2S^*, 1, 20, 20)$	0.1986 (0.0626)	-0.0001 (0.0049)	-0.0001 (0.0047)	-0.0001 (0.0006)	0.0001 (0.0049)	0.0001 (0.0051)
$NV(3S^*, 1, 20, 20)$	0.1983 (0.0671)	0.0000 (0.0034)	0.0000 (0.0033)	-0.0002 (0.0004)	0.0000 (0.0033)	-0.0000 (0.0035)

Table 7 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0010	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0020	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	3.4346 (0.1589)	3.3556 (0.2094)						
$RV(300)$	0.7669 (0.1238)	0.8210 (0.1373)						
$RV(900)$	0.6477 (0.1812)	0.5929 (0.1677)						
$RV(1800)$	0.6035 (0.2487)	0.5206 (0.2057)						
$RV_1(5)$	2.5892 (0.1429)	2.8902 (0.1979)						
$RV_1(300)$	0.6117 (0.1867)	0.5088 (0.1728)						
$RV_1(900)$	0.6001 (0.3002)	0.4978 (0.2493)						
$RV_1(1800)$	0.5786 (0.4008)	0.4949 (0.3431)						
$RV(\delta^*)$	0.6956 (0.1532)	0.6421 (0.1635)	0.0018 (0.0001)			0.0024 (0.0002)		
$RV_1(\delta^*)$	0.6034 (0.2073)	0.5056 (0.2111)	0.0018 (0.0001)			0.0024 (0.0002)		
$RV_2(\delta^*)$	0.6129 (0.2674)	0.5139 (0.2582)	0.0018 (0.0001)			0.0024 (0.0002)		
$K^{TH2}(60)$	0.6113 (0.1629)	0.5057 (0.1542)	0.0018 (0.0001)			0.0024 (0.0002)		
$TSRV$	0.6159 (0.0670)	0.5018 (0.0839)	0.0018 (0.0001)			0.0024 (0.0002)		
$NV(S^*, 0)$	0.6237 (0.0652)	0.5110 (0.0857)	0.0010 (0.0001)			0.0020 (0.0001)		
$NV(2S^*, 0)$	0.6211 (0.0699)	0.5122 (0.0818)	0.0010 (0.0001)			0.0020 (0.0001)		
$NV(3S^*, 0)$	0.6197 (0.0774)	0.5121 (0.0887)	0.0010 (0.0001)			0.0020 (0.0002)		
$NV(S^*, 10)$	0.6222 (0.1024)	0.5109 (0.1270)	0.0010 (0.0001)	-0.0010 (0.0260)	-0.0066 (0.0488)	0.0020 (0.0003)	0.0052 (0.1049)	0.0005 (0.0829)
$NV(2S^*, 10)$	0.6189 (0.0878)	0.5131 (0.0956)	0.0010 (0.0001)	0.0005 (0.0235)	0.0013 (0.0206)	0.0020 (0.0002)	0.0046 (0.0953)	-0.0005 (0.0776)
$NV(3S^*, 10)$	0.6180 (0.0930)	0.5125 (0.1015)	0.0010 (0.0001)	0.0042 (0.0359)	0.0295 (0.1026)	0.0020 (0.0003)	0.0037 (0.1099)	0.0000 (0.0826)
$NV(S^*, 20)$	0.6222 (0.1024)	0.5109 (0.1270)	0.0010 (0.0001)	-0.0010 (0.0260)	-0.0066 (0.0488)	0.0020 (0.0003)	0.0052 (0.1049)	0.0005 (0.0829)
$NV(2S^*, 20)$	0.6189 (0.0878)	0.5131 (0.0956)	0.0010 (0.0001)	0.0005 (0.0235)	0.0013 (0.0206)	0.0020 (0.0002)	0.0046 (0.0953)	-0.0005 (0.0776)
$NV(3S^*, 20)$	0.6180 (0.0930)	0.5125 (0.1015)	0.0010 (0.0001)	0.0042 (0.0359)	0.0295 (0.1026)	0.0020 (0.0003)	0.0037 (0.1099)	0.0000 (0.0826)

Table 8: Monte Carlo simulation results for scenario: **iid, mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0001	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.0140 (0.0505)					
$RC(300)$	0.1773 (0.0963)					
$RC(900)$	0.1791 (0.1350)					
$RC(1800)$	0.1775 (0.1736)					
$RC_{1,1}(5)$	0.0512 (0.0762)					
$RC_{1,1}(300)$	0.1926 (0.1361)					
$RC_{1,1}(900)$	0.1889 (0.2091)					
$RC_{1,1}(1800)$	0.1905 (0.2915)					
$RC(\delta^*)$	0.1132 (0.1010)			0.0001 (0.0001)		
$RC_{1,1}(\delta^*)$	0.1672 (0.1030)			0.0001 (0.0001)		
$RC_{2,2}(\delta^*)$	0.1804 (0.1097)			0.0001 (0.0001)		
HY	0.1811 (0.0938)					
$HY(S^*)$	0.1901 (0.0626)					
$NV(S^*, 1, 0, 0)$	0.1998 (0.0714)			-0.0002 (0.0010)		
$NV(2S^*, 1, 0, 0)$	0.1985 (0.0672)			-0.0001 (0.0009)		
$NV(3S^*, 1, 0, 0)$	0.1981 (0.0717)			-0.0001 (0.0011)		
$NV(S^*, 1, 10, 10)$	0.2002 (0.0824)	-0.0001 (0.0289)	-0.0003 (0.0182)	-0.0001 (0.0067)	0.0001 (0.0089)	0.0003 (0.0163)
$NV(2S^*, 1, 10, 10)$	0.1981 (0.0756)	-0.0004 (0.0165)	0.0001 (0.0141)	-0.0001 (0.0060)	0.0002 (0.0158)	0.0001 (0.0146)
$NV(3S^*, 1, 10, 10)$	0.1979 (0.0779)	-0.0000 (0.0061)	-0.0000 (0.0060)	-0.0001 (0.0026)	-0.0002 (0.0060)	0.0001 (0.0077)
$NV(S^*, 1, 20, 20)$	0.2002 (0.0824)	-0.0001 (0.0289)	-0.0003 (0.0182)	-0.0001 (0.0067)	0.0001 (0.0089)	0.0003 (0.0163)
$NV(2S^*, 1, 20, 20)$	0.2037 (0.1438)	-0.0106 (0.1823)	-0.0064 (0.1090)	0.0059 (0.1022)	0.0069 (0.1081)	0.0070 (0.1146)
$NV(3S^*, 1, 20, 20)$	0.1974 (0.0865)	0.0005 (0.0176)	-0.0002 (0.0127)	-0.0001 (0.0038)	-0.0002 (0.0121)	0.0002 (0.0165)

Table 8 ***(cont'd)*:** Monte Carlo simulation results for scenario: **iid, mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6265	0.5148	0.0100	0.0000	0.0000	0.0200	0.0000	0.0000
$RV(5)$	91.5016	141.7157						
	(2.2836)	(4.1276)						
$RV(300)$	2.1634	3.5872						
	(0.3985)	(0.6494)						
$RV(900)$	1.0906	1.4651						
	(0.3206)	(0.4413)						
$RV(1800)$	0.8214	0.9440						
	(0.3426)	(0.3913)						
$RV_1(5)$	3.2771	34.5228						
	(1.8989)	(3.1807)						
$RV_1(300)$	0.6160	0.5458						
	(0.3735)	(0.5701)						
$RV_1(900)$	0.6381	0.5424						
	(0.3892)	(0.4672)						
$RV_1(1800)$	0.5906	0.5226						
	(0.4621)	(0.4630)						
$RV(\delta^*)$	1.0071	1.1022	0.0108			0.0207		
	(0.3489)	(0.4382)	(0.0002)			(0.0005)		
$RV_1(\delta^*)$	0.6159	0.5478	0.0108			0.0207		
	(0.4117)	(0.4615)	(0.0002)			(0.0005)		
$RV_2(\delta^*)$	0.6388	0.5397	0.0108			0.0207		
	(0.4747)	(0.5122)	(0.0002)			(0.0005)		
$K^{TH2}(60)$	0.6144	0.5041	0.0108			0.0207		
	(0.2028)	(0.2199)	(0.0002)			(0.0005)		
$TSRV$	0.6054	0.4894	0.0108			0.0207		
	(0.0802)	(0.1196)	(0.0002)			(0.0005)		
$NV(S^*, 0)$	0.6225	0.5138	0.0100			0.0200		
	(0.0670)	(0.0923)	(0.0001)			(0.0004)		
$NV(2S^*, 0)$	0.6208	0.5126	0.0100			0.0200		
	(0.0712)	(0.0848)	(0.0001)			(0.0004)		
$NV(3S^*, 0)$	0.6192	0.5110	0.0100			0.0200		
	(0.0813)	(0.0932)	(0.0001)			(0.0004)		
$NV(S^*, 10)$	0.6239	0.5137	0.0100	-0.0000	0.0001	0.0200	-0.0004	-0.0015
	(0.0739)	(0.0964)	(0.0001)	(0.0006)	(0.0027)	(0.0004)	(0.0415)	(0.1188)
$NV(2S^*, 10)$	0.6205	0.5122	0.0100	-0.0000	0.0001	0.0200	-0.0005	-0.0008
	(0.0804)	(0.0917)	(0.0001)	(0.0006)	(0.0027)	(0.0004)	(0.0411)	(0.1150)
$NV(3S^*, 10)$	0.6185	0.5103	0.0100	-0.0000	0.0001	0.0200	-0.0004	-0.0009
	(0.0910)	(0.1012)	(0.0002)	(0.0006)	(0.0027)	(0.0004)	(0.0406)	(0.1119)
$NV(S^*, 20)$	0.6240	0.5179	0.0100	-0.0000	0.0001	0.0200	-0.0005	0.0004
	(0.0799)	(0.1023)	(0.0002)	(0.0006)	(0.0032)	(0.0004)	(0.0584)	(0.1712)
$NV(2S^*, 20)$	0.6200	0.5131	0.0100	-0.0000	0.0001	0.0200	-0.0007	0.0012
	(0.0850)	(0.0955)	(0.0002)	(0.0006)	(0.0032)	(0.0004)	(0.0557)	(0.1578)
$NV(3S^*, 20)$	0.6180	0.5106	0.0100	-0.0000	0.0001	0.0200	-0.0005	0.0008
	(0.0955)	(0.1051)	(0.0002)	(0.0006)	(0.0032)	(0.0004)	(0.0539)	(0.1487)

Table 9: Monte Carlo simulation results for scenario: **iid, high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0014	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	-2.1681					
	(1.9966)					
$RC(300)$	0.1541					
	(0.3472)					
$RC(900)$	0.1701					
	(0.2719)					
$RC(1800)$	0.1778					
	(0.2695)					
$RC_{1,1}(5)$	-0.1375					
	(1.6998)					
$RC_{1,1}(300)$	0.1854					
	(0.3158)					
$RC_{1,1}(900)$	0.1926					
	(0.3045)					
$RC_{1,1}(1800)$	0.2002					
	(0.3529)					
$RC(\delta^*)$	0.0553			-0.0002		
	(0.6652)			(0.0002)		
$RC_{1,1}(\delta^*)$	0.1928			-0.0002		
	(0.5557)			(0.0002)		
$RC_{2,2}(\delta^*)$	0.1949			-0.0002		
	(0.5735)			(0.0002)		
HY	-7.9298					
	(2.1305)					
$HY(S^*)$	-0.6376					
	(0.1917)					
$NV(S^*,1,0,0)$	0.1991			-0.0014		
	(0.0651)			(0.0003)		
$NV(2S^*,1,0,0)$	0.1985			-0.0014		
	(0.0625)			(0.0003)		
$NV(3S^*,1,0,0)$	0.1980			-0.0014		
	(0.0707)			(0.0003)		
$NV(S^*,1,10,10)$	0.1994	0.0008	0.0010	-0.0014	-0.0011	-0.0007
	(0.0742)	(0.0420)	(0.0410)	(0.0010)	(0.0412)	(0.0406)
$NV(2S^*,1,10,10)$	0.1987	0.0005	0.0007	-0.0014	-0.0007	-0.0005
	(0.0698)	(0.0248)	(0.0238)	(0.0007)	(0.0239)	(0.0233)
$NV(3S^*,1,10,10)$	0.1979	0.0001	0.0003	-0.0014	-0.0003	-0.0001
	(0.0783)	(0.0167)	(0.0158)	(0.0005)	(0.0160)	(0.0154)
$NV(S^*,1,20,20)$	0.1986	0.0014	0.0017	-0.0014	-0.0018	-0.0015
	(0.0752)	(0.0452)	(0.0444)	(0.0009)	(0.0447)	(0.0443)
$NV(2S^*,1,20,20)$	0.1985	0.0009	0.0011	-0.0014	-0.0011	-0.0009
	(0.0722)	(0.0206)	(0.0200)	(0.0006)	(0.0204)	(0.0200)
$NV(3S^*,1,20,20)$	0.1977	0.0003	0.0004	-0.0014	-0.0004	-0.0003
	(0.0814)	(0.0130)	(0.0126)	(0.0005)	(0.0130)	(0.0126)

Table 9 ***(cont'd)*:** Monte Carlo simulation results for scenario: **iid, high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6265	0.5148	0.0100	0.0000	0.0000	0.0200	0.0000	0.0000
$RV(5)$	64.2979	75.0218						
	(2.0523)	(3.0134)						
$RV(300)$	2.1226	3.6089						
	(0.3901)	(0.6663)						
$RV(900)$	1.0910	1.4852						
	(0.3020)	(0.4968)						
$RV(1800)$	0.8089	0.9740						
	(0.3209)	(0.4174)						
$RV_1(5)$	20.5938	44.7723						
	(1.5388)	(2.4782)						
$RV_1(300)$	0.6363	0.5415						
	(0.3637)	(0.5709)						
$RV_1(900)$	0.6279	0.5629						
	(0.3737)	(0.4587)						
$RV_1(1800)$	0.5974	0.5177						
	(0.4638)	(0.4668)						
$RV(\delta^*)$	0.9864	1.1072	0.0108			0.0206		
	(0.3361)	(0.4138)	(0.0003)			(0.0009)		
$RV_1(\delta^*)$	0.6294	0.5543	0.0108			0.0206		
	(0.4168)	(0.4746)	(0.0003)			(0.0009)		
$RV_2(\delta^*)$	0.6198	0.5793	0.0108			0.0206		
	(0.4746)	(0.5292)	(0.0003)			(0.0009)		
$K^{TH2}(60)$	0.6106	0.5103	0.0108			0.0206		
	(0.2144)	(0.2326)	(0.0003)			(0.0009)		
$TSRV$	0.6007	0.4814	0.0108			0.0206		
	(0.0954)	(0.1447)	(0.0003)			(0.0009)		
$NV(S^*, 0)$	0.6208	0.5127	0.0100			0.0200		
	(0.0823)	(0.1256)	(0.0002)			(0.0006)		
$NV(2S^*, 0)$	0.6187	0.5115	0.0100			0.0200		
	(0.0838)	(0.1046)	(0.0002)			(0.0006)		
$NV(3S^*, 0)$	0.6163	0.5082	0.0100			0.0200		
	(0.0961)	(0.1104)	(0.0002)			(0.0006)		
$NV(S^*, 10)$	0.6204	0.5111	0.0100	-0.0005	0.0054	0.0200	-0.0461	-0.1305
	(0.0946)	(0.1370)	(0.0003)	(0.0455)	(0.2289)	(0.0007)	(3.1430)	(6.2203)
$NV(2S^*, 10)$	0.6180	0.5104	0.0100	-0.0004	0.0039	0.0200	-0.0413	-0.1212
	(0.0950)	(0.1123)	(0.0003)	(0.0409)	(0.1969)	(0.0007)	(3.1002)	(6.2346)
$NV(3S^*, 10)$	0.6152	0.5067	0.0100	-0.0004	0.0024	0.0200	-0.0459	-0.1168
	(0.1079)	(0.1192)	(0.0003)	(0.0384)	(0.1820)	(0.0007)	(3.0991)	(6.2725)
$NV(S^*, 20)$	0.6174	0.5131	0.0100	-0.0027	0.0210	0.0200	0.4736	-0.3388
	(0.1000)	(0.1482)	(0.0003)	(0.1013)	(0.4408)	(0.0007)	(17.6227)	(13.4005)
$NV(2S^*, 20)$	0.6169	0.5109	0.0100	-0.0019	0.0154	0.0200	0.4897	-0.3229
	(0.0990)	(0.1170)	(0.0003)	(0.0962)	(0.3882)	(0.0007)	(17.4311)	(13.3083)
$NV(3S^*, 20)$	0.6144	0.5065	0.0100	-0.0016	0.0123	0.0200	0.4438	-0.4293
	(0.1121)	(0.1237)	(0.0003)	(0.0939)	(0.3702)	(0.0007)	(17.3720)	(13.3781)

Table 10: Monte Carlo simulation results for scenario: **iid, high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0014	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	-0.4430 (1.1277)					
$RC(300)$	0.1708 (0.3593)					
$RC(900)$	0.1860 (0.2859)					
$RC(1800)$	0.1839 (0.2808)					
$RC_{1,1}(5)$	-0.0301 (1.1977)					
$RC_{1,1}(300)$	0.1875 (0.3406)					
$RC_{1,1}(900)$	0.1983 (0.3023)					
$RC_{1,1}(1800)$	0.1910 (0.3519)					
$RC(\delta^*)$	0.1170 (0.6418)			-0.0001 (0.0003)		
$RC_{1,1}(\delta^*)$	0.1948 (0.5994)			-0.0001 (0.0003)		
$RC_{2,2}(\delta^*)$	0.1903 (0.5822)			-0.0001 (0.0003)		
HY	-1.0681 (1.1968)					
$HY(S^*)$	-0.0063 (0.1779)					
$NV(S^*, 1, 0, 0)$	0.1991 (0.0848)			-0.0014 (0.0012)		
$NV(2S^*, 1, 0, 0)$	0.1981 (0.0782)			-0.0014 (0.0011)		
$NV(3S^*, 1, 0, 0)$	0.1966 (0.0856)			-0.0014 (0.0012)		
$NV(S^*, 1, 10, 10)$	0.1980 (0.0921)	-0.0002 (0.0114)	-0.0003 (0.0119)	-0.0014 (0.0024)	0.0004 (0.0103)	0.0007 (0.0126)
$NV(2S^*, 1, 10, 10)$	0.1976 (0.0846)	-0.0002 (0.0072)	-0.0001 (0.0076)	-0.0014 (0.0016)	0.0002 (0.0067)	0.0004 (0.0079)
$NV(3S^*, 1, 10, 10)$	0.1960 (0.0922)	-0.0002 (0.0056)	-0.0001 (0.0058)	-0.0013 (0.0014)	0.0001 (0.0051)	0.0003 (0.0060)
$NV(S^*, 1, 20, 20)$	0.1993 (0.1019)	0.0001 (0.0218)	0.0000 (0.0210)	-0.0014 (0.0025)	0.0002 (0.0211)	0.0003 (0.0221)
$NV(2S^*, 1, 20, 20)$	0.1976 (0.0897)	-0.0002 (0.0128)	-0.0000 (0.0130)	-0.0014 (0.0016)	0.0002 (0.0122)	0.0003 (0.0131)
$NV(3S^*, 1, 20, 20)$	0.1958 (0.0971)	-0.0003 (0.0103)	-0.0002 (0.0102)	-0.0013 (0.0014)	0.0002 (0.0098)	0.0004 (0.0103)

Table 10 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0100	$\gamma_1(1)$ 0.0000	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0200	$\gamma_2(1)$ 0.0000	$\gamma_2(2)$ 0.0000
$RV(5)$	28.7542 (1.3773)	28.9485 (1.8435)						
$RV(300)$	2.1494 (0.3877)	3.6305 (0.6488)						
$RV(900)$	1.1020 (0.3291)	1.5065 (0.4724)						
$RV(1800)$	0.8161 (0.3442)	0.9423 (0.3891)						
$RV_1(5)$	20.2755 (1.2089)	24.2890 (1.7080)						
$RV_1(300)$	0.6231 (0.3842)	0.5288 (0.5941)						
$RV_1(900)$	0.6133 (0.3989)	0.5392 (0.4425)						
$RV_1(1800)$	0.5881 (0.4522)	0.5376 (0.4588)						
$RV(\delta^*)$	0.9999 (0.3626)	1.1622 (0.4349)	0.0107 (0.0006)			0.0179 (0.0013)		
$RV_1(\delta^*)$	0.6430 (0.4037)	0.5618 (0.4539)	0.0107 (0.0006)			0.0179 (0.0013)		
$RV_2(\delta^*)$	0.6231 (0.4691)	0.5591 (0.5352)	0.0107 (0.0006)			0.0179 (0.0013)		
$K^{TH2}(60)$	0.6111 (0.2302)	0.5124 (0.2432)	0.0107 (0.0006)			0.0179 (0.0013)		
$TSRV$	0.5981 (0.1199)	0.4758 (0.1882)	0.0107 (0.0006)			0.0179 (0.0013)		
$NV(S^*,0)$	0.6195 (0.1135)	0.5097 (0.1765)	0.0100 (0.0004)			0.0200 (0.0012)		
$NV(2S^*,0)$	0.6151 (0.1094)	0.5119 (0.1336)	0.0100 (0.0004)			0.0200 (0.0011)		
$NV(3S^*,0)$	0.6124 (0.1226)	0.5080 (0.1362)	0.0100 (0.0004)			0.0200 (0.0011)		
$NV(S^*,10)$	0.6181 (0.1328)	0.5177 (0.2096)	0.0100 (0.0005)	-0.0070 (0.1816)	-0.0026 (0.1463)	0.0200 (0.0014)	0.0372 (0.7121)	-0.0044 (0.6172)
$NV(2S^*,10)$	0.6131 (0.1249)	0.5143 (0.1451)	0.0100 (0.0005)	-0.0047 (0.1828)	-0.0011 (0.1475)	0.0200 (0.0012)	0.0287 (0.6676)	0.0001 (0.6011)
$NV(3S^*,10)$	0.6107 (0.1378)	0.5081 (0.1486)	0.0100 (0.0005)	-0.0029 (0.1875)	0.0001 (0.1501)	0.0200 (0.0012)	0.0123 (0.6741)	0.0088 (0.6018)
$NV(S^*,20)$	0.6181 (0.1328)	0.5177 (0.2096)	0.0100 (0.0005)	-0.0070 (0.1816)	-0.0026 (0.1463)	0.0200 (0.0014)	0.0372 (0.7121)	-0.0044 (0.6172)
$NV(2S^*,20)$	0.6131 (0.1249)	0.5143 (0.1451)	0.0100 (0.0005)	-0.0047 (0.1828)	-0.0011 (0.1475)	0.0200 (0.0012)	0.0287 (0.6676)	0.0001 (0.6011)
$NV(3S^*,20)$	0.6107 (0.1378)	0.5081 (0.1486)	0.0100 (0.0005)	-0.0029 (0.1875)	0.0001 (0.1501)	0.0200 (0.0012)	0.0123 (0.6741)	0.0088 (0.6018)

Table 11: Monte Carlo simulation results for scenario: **iid, high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ -0.0014	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	-0.0375 (0.4275)					
$RC(300)$	0.1703 (0.3776)					
$RC(900)$	0.1809 (0.2876)					
$RC(1800)$	0.1732 (0.2635)					
$RC_{1,1}(5)$	0.0115 (0.6371)					
$RC_{1,1}(300)$	0.1892 (0.3316)					
$RC_{1,1}(900)$	0.1829 (0.2983)					
$RC_{1,1}(1800)$	0.1893 (0.3553)					
$RC(\delta^*)$	0.1163 (0.5098)			0.0000 (0.0005)		
$RC_{1,1}(\delta^*)$	0.1776 (0.4881)			0.0000 (0.0005)		
$RC_{2,2}(\delta^*)$	0.1880 (0.4746)			0.0000 (0.0005)		
HY	0.0159 (0.6798)					
$HY(S^*)$	0.1450 (0.1895)					
$NV(S^*, 1, 0, 0)$	0.1979 (0.1285)			-0.0015 (0.0056)		
$NV(2S^*, 1, 0, 0)$	0.1981 (0.1033)			-0.0015 (0.0051)		
$NV(3S^*, 1, 0, 0)$	0.1967 (0.1056)			-0.0014 (0.0050)		
$NV(S^*, 1, 10, 10)$	0.1998 (0.1415)	-0.0001 (0.0333)	0.0007 (0.0326)	-0.0009 (0.0138)	-0.0021 (0.0324)	-0.0009 (0.0433)
$NV(2S^*, 1, 10, 10)$	0.1982 (0.1089)	-0.0001 (0.0144)	-0.0003 (0.0163)	-0.0013 (0.0065)	-0.0008 (0.0159)	0.0002 (0.0159)
$NV(3S^*, 1, 10, 10)$	0.1964 (0.1112)	-0.0002 (0.0113)	-0.0003 (0.0119)	-0.0013 (0.0052)	-0.0003 (0.0123)	-0.0000 (0.0120)
$NV(S^*, 1, 20, 20)$	0.1846 (0.3394)	0.0538 (0.9336)	-0.0113 (0.2581)	-0.0151 (0.1892)	0.0385 (0.5221)	0.0013 (0.2167)
$NV(2S^*, 1, 20, 20)$	0.1987 (0.1138)	0.0003 (0.0168)	-0.0004 (0.0183)	-0.0013 (0.0067)	-0.0007 (0.0170)	0.0000 (0.0173)
$NV(3S^*, 1, 20, 20)$	0.1965 (0.1159)	-0.0001 (0.0119)	-0.0003 (0.0122)	-0.0014 (0.0052)	-0.0001 (0.0123)	-0.0002 (0.0125)

Table 11 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0001	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0003	$\gamma_2(1)$ 0.0001	$\gamma_2(2)$ 0.0001
$RV(5)$	1.6962 (0.0370)	2.3318 (0.0621)						
$RV(300)$	0.6354 (0.1034)	0.5508 (0.0934)						
$RV(900)$	0.6051 (0.1658)	0.5051 (0.1383)						
$RV(1800)$	0.5842 (0.2377)	0.4815 (0.1926)						
$RV_1(5)$	0.7041 (0.0410)	1.0764 (0.0580)						
$RV_1(300)$	0.6116 (0.1738)	0.5078 (0.1437)						
$RV_1(900)$	0.6017 (0.2905)	0.4916 (0.2384)						
$RV_1(1800)$	0.5783 (0.3981)	0.4916 (0.3376)						
$RV(\delta^*)$	0.6274 (0.1087)	0.5366 (0.0988)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_1(\delta^*)$	0.6041 (0.1756)	0.4963 (0.1585)	0.0009 (0.0000)			0.0009 (0.0000)		
$RV_2(\delta^*)$	0.6063 (0.2153)	0.5002 (0.1865)	0.0009 (0.0000)			0.0009 (0.0000)		
$K^{TH2}(60)$	0.6131 (0.1291)	0.5066 (0.1122)	0.0009 (0.0000)			0.0009 (0.0000)		
$TSRV$	0.6746 (0.0330)	0.5801 (0.0378)	0.0009 (0.0000)			0.0009 (0.0000)		
$NV(S^*, 0)$	0.6786 (0.0286)	0.5896 (0.0322)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(2S^*, 0)$	0.6467 (0.0364)	0.5446 (0.0376)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(3S^*, 0)$	0.6372 (0.0426)	0.5315 (0.0428)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(S^*, 10)$	0.6235 (0.0422)	0.5131 (0.0480)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0000)	0.0003 (0.0000)	0.0002 (0.0007)	-0.0000 (0.0022)
$NV(2S^*, 10)$	0.6239 (0.0484)	0.5129 (0.0477)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0000)	0.0003 (0.0000)	0.0002 (0.0007)	-0.0000 (0.0023)
$NV(3S^*, 10)$	0.6232 (0.0538)	0.5123 (0.0523)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0001)	0.0003 (0.0000)	0.0002 (0.0008)	-0.0000 (0.0023)
$NV(S^*, 20)$	0.6247 (0.0764)	0.5158 (0.0978)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0001)	0.0003 (0.0001)	0.0002 (0.0013)	-0.0001 (0.0040)
$NV(2S^*, 20)$	0.6239 (0.0544)	0.5128 (0.0524)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0001)	0.0003 (0.0000)	0.0002 (0.0012)	-0.0000 (0.0039)
$NV(3S^*, 20)$	0.6229 (0.0591)	0.5121 (0.0566)	0.0001 (0.0000)	0.0001 (0.0000)	0.0000 (0.0001)	0.0003 (0.0000)	0.0002 (0.0013)	0.0000 (0.0041)

Table 12: Monte Carlo simulation results for scenario: **iid, low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0001	$\gamma_{12}(-1)$ 0.0001	$\gamma_{12}(0)$ 0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.1959 (0.0310)					
$RC(300)$	0.1984 (0.0747)					
$RC(900)$	0.1865 (0.1225)					
$RC(1800)$	0.1826 (0.1665)					
$RC_{1,1}(5)$	0.2445 (0.0336)					
$RC_{1,1}(300)$	0.1944 (0.1238)					
$RC_{1,1}(900)$	0.1925 (0.2028)					
$RC_{1,1}(1800)$	0.1931 (0.2868)					
$RC(\delta^*)$	0.2082 (0.0326)			0.0000 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1975 (0.0419)			0.0000 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1962 (0.0512)			0.0000 (0.0000)		
HY	0.2651 (0.0344)					
$HY(S^*)$	0.2448 (0.0210)					
$NV(S^*,1,0,0)$	0.2185 (0.0255)			0.0000 (0.0000)		
$NV(2S^*,1,0,0)$	0.2069 (0.0305)			0.0000 (0.0000)		
$NV(3S^*,1,0,0)$	0.2033 (0.0347)			0.0000 (0.0000)		
$NV(S^*,1,10,10)$	0.1994 (0.0384)	0.0000 (0.0022)	0.0001 (0.0022)	0.0000 (0.0001)	0.0000 (0.0022)	-0.0000 (0.0023)
$NV(2S^*,1,10,10)$	0.1985 (0.0389)	0.0001 (0.0017)	0.0001 (0.0017)	0.0000 (0.0001)	-0.0000 (0.0017)	-0.0000 (0.0018)
$NV(3S^*,1,10,10)$	0.1982 (0.0426)	0.0001 (0.0016)	0.0001 (0.0016)	0.0000 (0.0001)	-0.0000 (0.0016)	-0.0000 (0.0016)
$NV(S^*,1,20,20)$	0.1971 (0.1250)	0.0002 (0.0060)	0.0002 (0.0065)	0.0000 (0.0001)	-0.0001 (0.0062)	-0.0002 (0.0069)
$NV(2S^*,1,20,20)$	0.1983 (0.0429)	0.0002 (0.0035)	0.0002 (0.0037)	0.0000 (0.0001)	-0.0002 (0.0036)	-0.0001 (0.0037)
$NV(3S^*,1,20,20)$	0.1980 (0.0464)	0.0001 (0.0032)	0.0001 (0.0032)	0.0000 (0.0001)	-0.0001 (0.0032)	-0.0001 (0.0032)

Table 12 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **iid, low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0001	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0003	$\gamma_2(1)$ 0.0001	$\gamma_2(2)$ 0.0001
$RV(5)$	1.3767 (0.0388)	1.5045 (0.0579)						
$RV(300)$	0.6337 (0.1038)	0.5525 (0.0929)						
$RV(900)$	0.6058 (0.1646)	0.5051 (0.1405)						
$RV(1800)$	0.5841 (0.2350)	0.4820 (0.1934)						
$RV_1(5)$	0.8919 (0.0389)	1.1429 (0.0547)						
$RV_1(300)$	0.6119 (0.1724)	0.5083 (0.1451)						
$RV_1(900)$	0.6011 (0.2899)	0.4913 (0.2389)						
$RV_1(1800)$	0.5793 (0.3976)	0.4915 (0.3370)						
$RV(\delta^*)$	0.6247 (0.1084)	0.5364 (0.0998)	0.0009 (0.0000)			0.0009 (0.0001)		
$RV_1(\delta^*)$	0.5988 (0.1737)	0.4968 (0.1549)	0.0009 (0.0000)			0.0009 (0.0001)		
$RV_2(\delta^*)$	0.6066 (0.2130)	0.5022 (0.1866)	0.0009 (0.0000)			0.0009 (0.0001)		
$K^{TH2}(60)$	0.6121 (0.1350)	0.5055 (0.1164)	0.0009 (0.0000)			0.0009 (0.0001)		
$TSRV$	0.6421 (0.0394)	0.5368 (0.0438)	0.0009 (0.0000)			0.0009 (0.0001)		
$NV(S^*, 0)$	0.6457 (0.0368)	0.5446 (0.0412)	0.0001 (0.0000)			0.0002 (0.0000)		
$NV(2S^*, 0)$	0.6319 (0.0455)	0.5254 (0.0473)	0.0001 (0.0000)			0.0003 (0.0000)		
$NV(3S^*, 0)$	0.6270 (0.0522)	0.5197 (0.0538)	0.0001 (0.0000)			0.0003 (0.0000)		
$NV(S^*, 10)$	0.6232 (0.0554)	0.5140 (0.0862)	0.0001 (0.0000)	-0.0000 (0.0012)	0.0003 (0.0069)	0.0003 (0.0001)	-0.0026 (0.1104)	-0.0009 (0.1865)
$NV(2S^*, 10)$	0.6219 (0.0593)	0.5120 (0.0615)	0.0001 (0.0000)	-0.0000 (0.0013)	0.0003 (0.0071)	0.0003 (0.0001)	-0.0011 (0.0937)	0.0009 (0.1736)
$NV(3S^*, 10)$	0.6204 (0.0648)	0.5115 (0.0669)	0.0001 (0.0000)	-0.0001 (0.0014)	0.0004 (0.0072)	0.0003 (0.0001)	-0.0009 (0.0904)	0.0015 (0.1853)
$NV(S^*, 20)$	0.6182 (0.0963)	0.5130 (0.7430)	0.0001 (0.0001)	0.0000 (0.0034)	0.0007 (0.0181)	0.0003 (0.0010)	0.0090 (0.9513)	0.0097 (1.4487)
$NV(2S^*, 20)$	0.6212 (0.0641)	0.5119 (0.0682)	0.0001 (0.0000)	0.0000 (0.0031)	0.0005 (0.0163)	0.0003 (0.0001)	0.0070 (0.6326)	-0.0061 (0.4555)
$NV(3S^*, 20)$	0.6198 (0.0689)	0.5113 (0.0724)	0.0001 (0.0001)	0.0000 (0.0031)	0.0006 (0.0150)	0.0003 (0.0001)	0.0071 (0.6111)	-0.0079 (0.4572)

Table 13: Monte Carlo simulation results for scenario: **low pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0001	$\gamma_{12}(-1)$ 0.0001	$\gamma_{12}(0)$ 0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.0833 (0.0216)					
$RC(300)$	0.1934 (0.0757)					
$RC(900)$	0.1865 (0.1225)					
$RC(1800)$	0.1827 (0.1667)					
$RC_{1,1}(5)$	0.1581 (0.0297)					
$RC_{1,1}(300)$	0.1952 (0.1239)					
$RC_{1,1}(900)$	0.1910 (0.2013)					
$RC_{1,1}(1800)$	0.1928 (0.2852)					
$RC(\delta^*)$	0.1665 (0.0363)			0.0000 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1947 (0.0473)			0.0000 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1956 (0.0575)			0.0000 (0.0000)		
HY	0.2500 (0.0341)					
$HY(S^*)$	0.2238 (0.0286)					
$NV(S^*, 1, 0, 0)$	0.2031 (0.0351)			0.0001 (0.0000)		
$NV(2S^*, 1, 0, 0)$	0.2002 (0.0399)			0.0001 (0.0001)		
$NV(3S^*, 1, 0, 0)$	0.1993 (0.0455)			0.0001 (0.0001)		
$NV(S^*, 1, 10, 10)$	0.1980 (0.0482)	0.0002 (0.0038)	0.0001 (0.0037)	-0.0000 (0.0018)	0.0001 (0.0032)	-0.0001 (0.0055)
$NV(2S^*, 1, 10, 10)$	0.1981 (0.0478)	0.0000 (0.0014)	0.0000 (0.0012)	0.0000 (0.0004)	0.0000 (0.0013)	0.0000 (0.0013)
$NV(3S^*, 1, 10, 10)$	0.1980 (0.0533)	0.0000 (0.0009)	0.0000 (0.0009)	0.0000 (0.0003)	0.0001 (0.0009)	0.0000 (0.0011)
$NV(S^*, 1, 20, 20)$	0.1980 (0.0482)	0.0002 (0.0038)	0.0001 (0.0037)	-0.0000 (0.0018)	0.0001 (0.0032)	-0.0001 (0.0055)
$NV(2S^*, 1, 20, 20)$	0.1976 (0.0544)	0.0001 (0.0034)	0.0001 (0.0029)	0.0000 (0.0005)	-0.0001 (0.0038)	-0.0001 (0.0032)
$NV(3S^*, 1, 20, 20)$	0.1978 (0.0589)	-0.0000 (0.0021)	-0.0000 (0.0020)	0.0000 (0.0003)	0.0001 (0.0022)	0.0001 (0.0022)

Table 13 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0001	$\gamma_1(1)$ 0.0001	$\gamma_1(2)$ 0.0000	$\gamma_2(0)$ 0.0003	$\gamma_2(1)$ 0.0001	$\gamma_2(2)$ 0.0001
$RV(5)$	0.9642 (0.0434)	0.8992 (0.0559)						
$RV(300)$	0.6338 (0.1033)	0.5525 (0.0929)						
$RV(900)$	0.6039 (0.1679)	0.5055 (0.1399)						
$RV(1800)$	0.5833 (0.2389)	0.4812 (0.1925)						
$RV_1(5)$	0.8692 (0.0429)	0.8415 (0.0561)						
$RV_1(300)$	0.6117 (0.1736)	0.5077 (0.1439)						
$RV_1(900)$	0.5984 (0.2926)	0.4911 (0.2373)						
$RV_1(1800)$	0.5783 (0.3973)	0.4903 (0.3352)						
$RV(\delta^*)$	0.6235 (0.1095)	0.5387 (0.0982)	0.0009 (0.0001)			0.0009 (0.0001)		
$RV_1(\delta^*)$	0.5969 (0.1722)	0.4942 (0.1538)	0.0009 (0.0001)			0.0009 (0.0001)		
$RV_2(\delta^*)$	0.6076 (0.2127)	0.4984 (0.1863)	0.0009 (0.0001)			0.0009 (0.0001)		
$K^{TH2}(60)$	0.6124 (0.1459)	0.5065 (0.1261)	0.0009 (0.0001)			0.0009 (0.0001)		
$TSRV$	0.6278 (0.0523)	0.5171 (0.0636)	0.0009 (0.0001)			0.0009 (0.0001)		
$NV(S^*, 0)$	0.6330 (0.0499)	0.5232 (0.0626)	0.0001 (0.0000)			0.0003 (0.0000)		
$NV(2S^*, 0)$	0.6267 (0.0581)	0.5157 (0.0663)	0.0001 (0.0000)			0.0003 (0.0001)		
$NV(3S^*, 0)$	0.6236 (0.0658)	0.5143 (0.0728)	0.0001 (0.0000)			0.0003 (0.0001)		
$NV(S^*, 10)$	0.6193 (0.1472)	0.5126 (0.1015)	0.0001 (0.0002)	0.0006 (0.0149)	0.0003 (0.0117)	0.0003 (0.0002)	-0.0004 (0.0304)	0.0008 (0.0318)
$NV(2S^*, 10)$	0.6215 (0.0757)	0.5102 (0.0815)	0.0001 (0.0001)	0.0004 (0.0100)	-0.0007 (0.0359)	0.0003 (0.0001)	-0.0014 (0.0329)	0.0013 (0.0240)
$NV(3S^*, 10)$	0.6194 (0.0819)	0.5118 (0.0866)	0.0001 (0.0001)	0.0008 (0.0090)	-0.0001 (0.0071)	0.0003 (0.0001)	0.0003 (0.0449)	0.0006 (0.0287)
$NV(S^*, 20)$	0.6193 (0.1472)	0.5126 (0.1015)	0.0001 (0.0002)	0.0006 (0.0149)	0.0003 (0.0117)	0.0003 (0.0002)	-0.0004 (0.0304)	0.0008 (0.0318)
$NV(2S^*, 20)$	0.6215 (0.0757)	0.5102 (0.0815)	0.0001 (0.0001)	0.0004 (0.0100)	-0.0007 (0.0359)	0.0003 (0.0001)	-0.0014 (0.0329)	0.0013 (0.0240)
$NV(3S^*, 20)$	0.6194 (0.0819)	0.5118 (0.0866)	0.0001 (0.0001)	0.0008 (0.0090)	-0.0001 (0.0071)	0.0003 (0.0001)	0.0003 (0.0449)	0.0006 (0.0287)

Table 14: Monte Carlo simulation results for scenario: **low pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0001	$\gamma_{12}(-1)$ 0.0001	$\gamma_{12}(0)$ 0.0000	$\gamma_{12}(1)$ 0.0000	$\gamma_{12}(2)$ 0.0000
$RC(5)$	0.0247 (0.0140)					
$RC(300)$	0.1791 (0.0737)					
$RC(900)$	0.1815 (0.1223)					
$RC(1800)$	0.1796 (0.1662)					
$RC_{1,1}(5)$	0.0632 (0.0223)					
$RC_{1,1}(300)$	0.1945 (0.1217)					
$RC_{1,1}(900)$	0.1905 (0.2021)					
$RC_{1,1}(1800)$	0.1907 (0.2851)					
$RC(\delta^*)$	0.1320 (0.0447)			0.0001 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1857 (0.0631)			0.0001 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1910 (0.0760)			0.0001 (0.0000)		
HY	0.2087 (0.0430)					
$HY(S^*)$	0.2053 (0.0417)					
$NV(S^*, 1, 0, 0)$	0.2002 (0.0521)			0.0001 (0.0004)		
$NV(2S^*, 1, 0, 0)$	0.1990 (0.0573)			0.0001 (0.0005)		
$NV(3S^*, 1, 0, 0)$	0.1986 (0.0629)			0.0001 (0.0006)		
$NV(S^*, 1, 10, 10)$	0.2001 (0.0519)		0.0004 (0.0082)	0.0000 (0.0017)	-0.0003 (0.0052)	
$NV(2S^*, 1, 10, 10)$	0.1972 (0.0767)	0.0056 (0.0815)	-0.0055 (0.0991)	-0.0004 (0.0278)	0.0033 (0.1091)	0.0043 (0.0723)
$NV(3S^*, 1, 10, 10)$	0.1978 (0.0705)	-0.0002 (0.0052)	0.0001 (0.0042)	-0.0000 (0.0022)	0.0001 (0.0040)	0.0001 (0.0067)
$NV(S^*, 1, 20, 20)$	0.2001 (0.0519)		0.0004 (0.0082)	0.0000 (0.0017)	-0.0003 (0.0052)	
$NV(2S^*, 1, 20, 20)$	0.1972 (0.0767)	0.0056 (0.0815)	-0.0055 (0.0991)	-0.0004 (0.0278)	0.0033 (0.1091)	0.0043 (0.0723)
$NV(3S^*, 1, 20, 20)$	0.1938 (0.3337)	-0.0004 (0.0166)	0.0009 (0.0242)	0.0000 (0.0457)	0.0003 (0.1008)	-0.0011 (0.0386)

Table 14 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0012	$\gamma_1(1)$ 0.0005	$\gamma_1(2)$ 0.0002	$\gamma_2(0)$ 0.0028	$\gamma_2(1)$ 0.0014	$\gamma_2(2)$ 0.0008
$RV(5)$	11.3375 (0.2676)	18.6869 (0.5102)						
$RV(300)$	0.8101 (0.1326)	0.9358 (0.1622)						
$RV(900)$	0.6600 (0.1805)	0.6280 (0.1741)						
$RV(1800)$	0.6094 (0.2472)	0.5412 (0.2144)						
$RV_1(5)$	1.4234 (0.2388)	6.1324 (0.4478)						
$RV_1(300)$	0.6092 (0.1888)	0.5151 (0.1844)						
$RV_1(900)$	0.6053 (0.2992)	0.4974 (0.2625)						
$RV_1(1800)$	0.5802 (0.4047)	0.4919 (0.3485)						
$RV(\delta^*)$	0.7189 (0.1634)	0.6698 (0.1936)	0.0020 (0.0001)			0.0034 (0.0001)		
$RV_1(\delta^*)$	0.6051 (0.2245)	0.5059 (0.2329)	0.0020 (0.0001)			0.0034 (0.0001)		
$RV_2(\delta^*)$	0.6134 (0.2796)	0.5206 (0.2932)	0.0020 (0.0001)			0.0034 (0.0001)		
$K^{TH2}(60)$	0.6100 (0.1491)	0.5054 (0.1489)	0.0020 (0.0001)			0.0034 (0.0001)		
$TSRV$	0.9367 (0.0752)	0.8115 (0.0810)	0.0020 (0.0001)			0.0034 (0.0001)		
$NV(S^*, 0)$	0.9189 (0.0431)	0.7897 (0.0539)	0.0010 (0.0000)			0.0024 (0.0001)		
$NV(2S^*, 0)$	0.7464 (0.0472)	0.6270 (0.0548)	0.0011 (0.0000)			0.0025 (0.0001)		
$NV(3S^*, 0)$	0.6964 (0.0541)	0.5808 (0.0621)	0.0011 (0.0000)			0.0025 (0.0001)		
$NV(S^*, 10)$	0.6243 (0.0568)	0.5120 (0.0665)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0003)	0.0028 (0.0001)	0.0014 (0.0057)	0.0005 (0.0168)
$NV(2S^*, 10)$	0.6232 (0.0581)	0.5122 (0.0638)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0003)	0.0028 (0.0001)	0.0014 (0.0057)	0.0006 (0.0165)
$NV(3S^*, 10)$	0.6217 (0.0646)	0.5117 (0.0714)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0003)	0.0028 (0.0001)	0.0013 (0.0056)	0.0006 (0.0163)
$NV(S^*, 20)$	0.6246 (0.0710)	0.5139 (0.0749)	0.0012 (0.0001)	0.0005 (0.0001)	0.0002 (0.0004)	0.0028 (0.0001)	0.0014 (0.0085)	0.0008 (0.0255)
$NV(2S^*, 20)$	0.6228 (0.0633)	0.5125 (0.0679)	0.0012 (0.0000)	0.0005 (0.0001)	0.0002 (0.0004)	0.0028 (0.0001)	0.0014 (0.0082)	0.0008 (0.0243)
$NV(3S^*, 20)$	0.6212 (0.0695)	0.5118 (0.0754)	0.0012 (0.0001)	0.0005 (0.0001)	0.0002 (0.0004)	0.0028 (0.0001)	0.0014 (0.0081)	0.0010 (0.0237)

Table 15: Monte Carlo simulation results for scenario: **low pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0005	$\gamma_{12}(-1)$ 0.0006	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0001
$RC(5)$	0.9552 (0.2462)					
$RC(300)$	0.2243 (0.1068)					
$RC(900)$	0.1947 (0.1412)					
$RC(1800)$	0.1880 (0.1793)					
$RC_{1,1}(5)$	0.8438 (0.2262)					
$RC_{1,1}(300)$	0.1942 (0.1437)					
$RC_{1,1}(900)$	0.1940 (0.2161)					
$RC_{1,1}(1800)$	0.1939 (0.2955)					
$RC(\delta^*)$	0.3128 (0.1236)			0.0001 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1988 (0.1139)			0.0001 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1974 (0.1220)			0.0001 (0.0000)		
HY	0.8571 (0.2543)					
$HY(S^*)$	0.5123 (0.0574)					
$NV(S^*, 1, 0, 0)$	0.2863 (0.0404)			0.0002 (0.0000)		
$NV(2S^*, 1, 0, 0)$	0.2349 (0.0414)			0.0002 (0.0001)		
$NV(3S^*, 1, 0, 0)$	0.2204 (0.0463)			0.0003 (0.0001)		
$NV(S^*, 1, 10, 10)$	0.1989 (0.0530)	0.0003 (0.0086)	0.0004 (0.0085)	0.0001 (0.0002)	0.0002 (0.0086)	0.0003 (0.0087)
$NV(2S^*, 1, 10, 10)$	0.1982 (0.0489)	0.0003 (0.0064)	0.0005 (0.0063)	0.0001 (0.0002)	0.0002 (0.0063)	0.0002 (0.0063)
$NV(3S^*, 1, 10, 10)$	0.1982 (0.0538)	0.0003 (0.0052)	0.0004 (0.0051)	0.0001 (0.0001)	0.0003 (0.0052)	0.0003 (0.0051)
$NV(S^*, 1, 20, 20)$	0.1982 (0.0635)	0.0006 (0.0143)	0.0008 (0.0147)	0.0001 (0.0003)	-0.0001 (0.0145)	-0.0001 (0.0148)
$NV(2S^*, 1, 20, 20)$	0.1981 (0.0522)	0.0004 (0.0088)	0.0005 (0.0088)	0.0001 (0.0002)	0.0001 (0.0088)	0.0001 (0.0087)
$NV(3S^*, 1, 20, 20)$	0.1982 (0.0572)	0.0003 (0.0062)	0.0004 (0.0062)	0.0001 (0.0001)	0.0003 (0.0062)	0.0003 (0.0061)

Table 15 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0012	$\gamma_1(1)$ 0.0005	$\gamma_1(2)$ 0.0002	$\gamma_2(0)$ 0.0028	$\gamma_2(1)$ 0.0014	$\gamma_2(2)$ 0.0008
$RV(5)$	8.1530	10.4012						
	(0.2501)	(0.4162)						
$RV(300)$	0.8030	0.9414						
	(0.1331)	(0.1647)						
$RV(900)$	0.6623	0.6291						
	(0.1763)	(0.1816)						
$RV(1800)$	0.6081	0.5435						
	(0.2406)	(0.2176)						
$RV_1(5)$	3.2935	6.7901						
	(0.1951)	(0.3614)						
$RV_1(300)$	0.6136	0.5156						
	(0.1870)	(0.1892)						
$RV_1(900)$	0.6032	0.4987						
	(0.2964)	(0.2601)						
$RV_1(1800)$	0.5820	0.4922						
	(0.4047)	(0.3489)						
$RV(\delta^*)$	0.7208	0.6654	0.0020			0.0034		
	(0.1635)	(0.1867)	(0.0001)			(0.0002)		
$RV_1(\delta^*)$	0.6002	0.5087	0.0020			0.0034		
	(0.2231)	(0.2321)	(0.0001)			(0.0002)		
$RV_2(\delta^*)$	0.6125	0.5135	0.0020			0.0034		
	(0.2774)	(0.2930)	(0.0001)			(0.0002)		
$K^{TH2}(60)$	0.6097	0.5051	0.0020			0.0034		
	(0.1550)	(0.1521)	(0.0001)			(0.0002)		
$TSRV$	0.7374	0.6140	0.0020			0.0034		
	(0.0627)	(0.0774)	(0.0001)			(0.0002)		
$NV(S^*, 0)$	0.7438	0.6228	0.0011			0.0026		
	(0.0520)	(0.0655)	(0.0000)			(0.0001)		
$NV(2S^*, 0)$	0.6706	0.5566	0.0012			0.0026		
	(0.0558)	(0.0675)	(0.0000)			(0.0001)		
$NV(3S^*, 0)$	0.6496	0.5377	0.0012			0.0026		
	(0.0641)	(0.0754)	(0.0000)			(0.0001)		
$NV(S^*, 10)$	0.6237	0.5127	0.0012	0.0001	0.0026	0.0028	-0.0084	0.0075
	(0.0718)	(0.0881)	(0.0001)	(0.0062)	(0.0339)	(0.0002)	(0.5006)	(0.9612)
$NV(2S^*, 10)$	0.6204	0.5115	0.0012	0.0000	0.0026	0.0028	-0.0041	0.0056
	(0.0685)	(0.0800)	(0.0001)	(0.0060)	(0.0315)	(0.0001)	(0.4781)	(0.9665)
$NV(3S^*, 10)$	0.6192	0.5107	0.0012	0.0000	0.0021	0.0028	-0.0014	0.0074
	(0.0765)	(0.0874)	(0.0001)	(0.0059)	(0.0302)	(0.0002)	(0.4723)	(0.9734)
$NV(S^*, 20)$	0.6220	0.5134	0.0012	-0.0002	0.0046	0.0028	0.0795	0.0003
	(0.0895)	(0.1050)	(0.0001)	(0.0154)	(0.0739)	(0.0002)	(2.8491)	(2.1677)
$NV(2S^*, 20)$	0.6194	0.5114	0.0012	-0.0001	0.0040	0.0028	0.0683	-0.0014
	(0.0729)	(0.0850)	(0.0001)	(0.0144)	(0.0636)	(0.0002)	(2.6857)	(2.0900)
$NV(3S^*, 20)$	0.6186	0.5105	0.0012	0.0001	0.0030	0.0028	0.0673	-0.0029
	(0.0807)	(0.0922)	(0.0001)	(0.0141)	(0.0595)	(0.0002)	(2.6563)	(2.1314)

Table 16: Monte Carlo simulation results for scenario: **low pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0005	$\gamma_{12}(-1)$ 0.0006	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0001
$RC(5)$	0.3400 (0.1459)					
$RC(300)$	0.2103 (0.1086)					
$RC(900)$	0.1947 (0.1421)					
$RC(1800)$	0.1876 (0.1805)					
$RC_{1,1}(5)$	0.4731 (0.1670)					
$RC_{1,1}(300)$	0.1946 (0.1476)					
$RC_{1,1}(900)$	0.1910 (0.2111)					
$RC_{1,1}(1800)$	0.1937 (0.2923)					
$RC(\delta^*)$	0.2440 (0.1149)			0.0001 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1999 (0.1106)			0.0001 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1949 (0.1207)			0.0001 (0.0000)		
HY	0.7078 (0.1682)					
$HY(S^*)$	0.3811 (0.0592)					
$NV(S^*, 1, 0, 0)$	0.2219 (0.0502)			0.0006 (0.0002)		
$NV(2S^*, 1, 0, 0)$	0.2087 (0.0511)			0.0006 (0.0002)		
$NV(3S^*, 1, 0, 0)$	0.2048 (0.0571)			0.0006 (0.0002)		
$NV(S^*, 1, 10, 10)$	0.1985 (0.0618)	0.0003 (0.0052)	0.0003 (0.0044)	0.0000 (0.0014)	0.0005 (0.0049)	0.0005 (0.0053)
$NV(2S^*, 1, 10, 10)$	0.1984 (0.0583)	0.0002 (0.0024)	0.0002 (0.0026)	0.0001 (0.0006)	0.0004 (0.0023)	0.0004 (0.0029)
$NV(3S^*, 1, 10, 10)$	0.1983 (0.0641)	0.0002 (0.0020)	0.0003 (0.0020)	0.0001 (0.0005)	0.0004 (0.0019)	0.0003 (0.0022)
$NV(S^*, 1, 20, 20)$	0.1993 (0.0838)	0.0016 (0.0243)	0.0014 (0.0227)	0.0002 (0.0023)	-0.0015 (0.0298)	-0.0006 (0.0268)
$NV(2S^*, 1, 20, 20)$	0.1984 (0.0644)	0.0003 (0.0050)	0.0004 (0.0049)	0.0001 (0.0006)	0.0003 (0.0050)	0.0002 (0.0053)
$NV(3S^*, 1, 20, 20)$	0.1981 (0.0694)	0.0003 (0.0039)	0.0004 (0.0037)	0.0001 (0.0005)	0.0003 (0.0038)	0.0002 (0.0040)

Table 16 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0012	$\gamma_1(1)$ 0.0005	$\gamma_1(2)$ 0.0002	$\gamma_2(0)$ 0.0028	$\gamma_2(1)$ 0.0014	$\gamma_2(2)$ 0.0008
$RV(5)$	4.0286 (0.1924)	4.3784 (0.2730)						
$RV(300)$	0.8059 (0.1312)	0.9426 (0.1624)						
$RV(900)$	0.6603 (0.1852)	0.6320 (0.1796)						
$RV(1800)$	0.6083 (0.2501)	0.5379 (0.2139)						
$RV_1(5)$	3.0690 (0.1716)	3.8014 (0.2582)						
$RV_1(300)$	0.6123 (0.1904)	0.5098 (0.1887)						
$RV_1(900)$	0.5982 (0.3033)	0.4968 (0.2548)						
$RV_1(1800)$	0.5794 (0.4016)	0.4956 (0.3449)						
$RV(\delta^*)$	0.7158 (0.1630)	0.6818 (0.1807)	0.0020 (0.0001)			0.0030 (0.0002)		
$RV_1(\delta^*)$	0.6049 (0.2260)	0.5133 (0.2366)	0.0020 (0.0001)			0.0030 (0.0002)		
$RV_2(\delta^*)$	0.6169 (0.2780)	0.5113 (0.2786)	0.0020 (0.0001)			0.0030 (0.0002)		
$K^{TH2}(60)$	0.6120 (0.1663)	0.5063 (0.1623)	0.0020 (0.0001)			0.0030 (0.0002)		
$TSRV$	0.6502 (0.0727)	0.5334 (0.0979)	0.0020 (0.0001)			0.0030 (0.0002)		
$NV(S^*, 0)$	0.6626 (0.0683)	0.5445 (0.0914)	0.0012 (0.0001)			0.0027 (0.0002)		
$NV(2S^*, 0)$	0.6366 (0.0734)	0.5260 (0.0875)	0.0012 (0.0001)			0.0027 (0.0002)		
$NV(3S^*, 0)$	0.6285 (0.0816)	0.5196 (0.0954)	0.0012 (0.0001)			0.0027 (0.0002)		
$NV(S^*, 10)$	0.6218 (0.0990)	0.5071 (0.1236)	0.0012 (0.0001)	0.0005 (0.0280)	0.0001 (0.0229)	0.0028 (0.0003)	0.0008 (0.1326)	0.0060 (0.1021)
$NV(2S^*, 10)$	0.6189 (0.0905)	0.5132 (0.1017)	0.0012 (0.0001)	0.0006 (0.0291)	-0.0037 (0.0635)	0.0028 (0.0003)	0.0067 (0.1250)	0.0029 (0.0996)
$NV(3S^*, 10)$	0.6174 (0.0971)	0.5117 (0.1087)	0.0012 (0.0002)	0.0007 (0.0307)	-0.0070 (0.0597)	0.0028 (0.0003)	0.0040 (0.1428)	0.0043 (0.1058)
$NV(S^*, 20)$	0.6218 (0.0990)	0.5071 (0.1236)	0.0012 (0.0001)	0.0005 (0.0280)	0.0001 (0.0229)	0.0028 (0.0003)	0.0008 (0.1326)	0.0060 (0.1021)
$NV(2S^*, 20)$	0.6189 (0.0905)	0.5132 (0.1017)	0.0012 (0.0001)	0.0006 (0.0291)	-0.0037 (0.0635)	0.0028 (0.0003)	0.0067 (0.1250)	0.0029 (0.0996)
$NV(3S^*, 20)$	0.6174 (0.0971)	0.5117 (0.1087)	0.0012 (0.0002)	0.0007 (0.0307)	-0.0070 (0.0597)	0.0028 (0.0003)	0.0040 (0.1428)	0.0043 (0.1058)

Table 17: Monte Carlo simulation results for scenario: **low pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0005	$\gamma_{12}(-1)$ 0.0006	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0001
$RC(5)$	0.0669 (0.0629)					
$RC(300)$	0.1859 (0.1070)					
$RC(900)$	0.1839 (0.1413)					
$RC(1800)$	0.1795 (0.1787)					
$RC_{1,1}(5)$	0.1326 (0.0945)					
$RC_{1,1}(300)$	0.1932 (0.1428)					
$RC_{1,1}(900)$	0.1895 (0.2137)					
$RC_{1,1}(1800)$	0.1902 (0.2920)					
$RC(\delta^*)$	0.1498 (0.1056)			0.0001 (0.0001)		
$RC_{1,1}(\delta^*)$	0.1782 (0.1129)			0.0001 (0.0001)		
$RC_{2,2}(\delta^*)$	0.1897 (0.1289)			0.0001 (0.0001)		
HY	0.2947 (0.1157)					
$HY(S^*)$	0.2457 (0.0677)					
$NV(S^*, 1, 0, 0)$	0.2044 (0.0714)			0.0008 (0.0011)		
$NV(2S^*, 1, 0, 0)$	0.2009 (0.0699)			0.0008 (0.0011)		
$NV(3S^*, 1, 0, 0)$	0.2002 (0.0756)			0.0008 (0.0013)		
$NV(S^*, 1, 10, 10)$	0.1998 (0.0814)	0.0018 (0.0340)	-0.0005 (0.0191)	-0.0000 (0.0118)	0.0016 (0.0201)	0.0003 (0.0142)
$NV(2S^*, 1, 10, 10)$	0.1976 (0.0772)	0.0000 (0.0085)	0.0003 (0.0083)	0.0001 (0.0033)	0.0002 (0.0080)	0.0002 (0.0110)
$NV(3S^*, 1, 10, 10)$	0.1977 (0.0817)	0.0003 (0.0057)	0.0002 (0.0055)	0.0001 (0.0022)	0.0001 (0.0056)	0.0003 (0.0056)
$NV(S^*, 1, 20, 20)$	0.1998 (0.0814)	0.0018 (0.0340)	-0.0005 (0.0191)	-0.0000 (0.0118)	0.0016 (0.0201)	0.0003 (0.0142)
$NV(2S^*, 1, 20, 20)$	0.1951 (0.1125)	0.0167 (0.2018)	0.0012 (0.0546)	-0.0032 (0.0400)	0.0064 (0.1083)	-0.0024 (0.0462)
$NV(3S^*, 1, 20, 20)$	0.1977 (0.0878)	0.0003 (0.0096)	0.0000 (0.0074)	0.0000 (0.0026)	0.0002 (0.0070)	0.0003 (0.0072)

Table 17 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0124	$\gamma_1(1)$ 0.0052	$\gamma_1(2)$ 0.0024	$\gamma_2(0)$ 0.0277	$\gamma_2(1)$ 0.0139	$\gamma_2(2)$ 0.0076
$RV(5)$	107.7640	182.2427						
	(2.6030)	(5.0116)						
$RV(300)$	2.5510	4.7802						
	(0.4699)	(0.8571)						
$RV(900)$	1.2133	1.8704						
	(0.3520)	(0.5809)						
$RV(1800)$	0.8745	1.1443						
	(0.3623)	(0.4873)						
$RV_1(5)$	8.6348	56.6998						
	(2.2711)	(4.3700)						
$RV_1(300)$	0.6026	0.5749						
	(0.4228)	(0.7364)						
$RV_1(900)$	0.6366	0.5518						
	(0.4195)	(0.5683)						
$RV_1(1800)$	0.6002	0.5148						
	(0.4790)	(0.5202)						
$RV(\delta^*)$	1.0579	1.2637	0.0132			0.0283		
	(0.3760)	(0.5403)	(0.0003)			(0.0008)		
$RV_1(\delta^*)$	0.6247	0.5707	0.0132			0.0283		
	(0.4294)	(0.5255)	(0.0003)			(0.0008)		
$RV_2(\delta^*)$	0.6311	0.5610	0.0132			0.0283		
	(0.4992)	(0.5894)	(0.0003)			(0.0008)		
$K^{TH2}(60)$	0.6157	0.4981	0.0132			0.0283		
	(0.2131)	(0.2448)	(0.0003)			(0.0008)		
$TSRV$	1.8641	1.8606	0.0132			0.0283		
	(0.1753)	(0.1883)	(0.0003)			(0.0008)		
$NV(S^*, 0)$	1.5913	1.6004	0.0109			0.0251		
	(0.0891)	(0.1333)	(0.0002)			(0.0005)		
$NV(2S^*, 0)$	1.0299	0.9678	0.0112			0.0256		
	(0.0806)	(0.1010)	(0.0002)			(0.0005)		
$NV(3S^*, 0)$	0.8674	0.7867	0.0113			0.0258		
	(0.0880)	(0.1037)	(0.0002)			(0.0005)		
$NV(S^*, 10)$	0.6234	0.5158	0.0123	0.0051	0.0024	0.0277	0.0129	0.0079
	(0.0931)	(0.1361)	(0.0002)	(0.0006)	(0.0028)	(0.0006)	(0.0549)	(0.1584)
$NV(2S^*, 10)$	0.6196	0.5123	0.0124	0.0051	0.0024	0.0277	0.0128	0.0088
	(0.0880)	(0.1061)	(0.0002)	(0.0006)	(0.0028)	(0.0006)	(0.0541)	(0.1521)
$NV(3S^*, 10)$	0.6172	0.5103	0.0124	0.0051	0.0024	0.0277	0.0128	0.0089
	(0.0968)	(0.1110)	(0.0002)	(0.0006)	(0.0028)	(0.0006)	(0.0532)	(0.1469)
$NV(S^*, 20)$	0.6227	0.5211	0.0123	0.0051	0.0023	0.0276	0.0130	0.0107
	(0.0988)	(0.1414)	(0.0002)	(0.0007)	(0.0034)	(0.0006)	(0.0776)	(0.2261)
$NV(2S^*, 20)$	0.6189	0.5133	0.0124	0.0051	0.0023	0.0277	0.0127	0.0115
	(0.0919)	(0.1093)	(0.0002)	(0.0007)	(0.0034)	(0.0006)	(0.0737)	(0.2069)
$NV(3S^*, 20)$	0.6165	0.5106	0.0124	0.0051	0.0023	0.0277	0.0128	0.0112
	(0.1009)	(0.1147)	(0.0002)	(0.0007)	(0.0033)	(0.0006)	(0.0710)	(0.1935)

Table 18: Monte Carlo simulation results for scenario: **low pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0052	$\gamma_{12}(-1)$ 0.0060	$\gamma_{12}(0)$ 0.0012	$\gamma_{12}(1)$ 0.0018	$\gamma_{12}(2)$ 0.0014
$RC(5)$	8.5317 (2.4038)					
$RC(300)$	0.4642 (0.4481)					
$RC(900)$	0.2791 (0.3394)					
$RC(1800)$	0.2380 (0.3098)					
$RC_{1,1}(5)$	6.8537 (2.1534)					
$RC_{1,1}(300)$	0.1936 (0.3957)					
$RC_{1,1}(900)$	0.1960 (0.3623)					
$RC_{1,1}(1800)$	0.1982 (0.3791)					
$RC(\delta^*)$	0.6782 (0.6188)			0.0006 (0.0002)		
$RC_{1,1}(\delta^*)$	0.1618 (0.5069)			0.0006 (0.0002)		
$RC_{2,2}(\delta^*)$	0.2351 (0.4945)			0.0006 (0.0002)		
HY	6.7653 (2.4644)					
$HY(S^*)$	1.9360 (0.2549)					
$NV(S^*, 1, 0, 0)$	0.4946 (0.0846)			0.0025 (0.0004)		
$NV(2S^*, 1, 0, 0)$	0.3252 (0.0708)			0.0028 (0.0004)		
$NV(3S^*, 1, 0, 0)$	0.2762 (0.0765)			0.0029 (0.0004)		
$NV(S^*, 1, 10, 10)$	0.1996 (0.0918)	0.0036 (0.0477)	0.0044 (0.0466)	0.0012 (0.0011)	0.0021 (0.0467)	0.0017 (0.0462)
$NV(2S^*, 1, 10, 10)$	0.1989 (0.0776)	0.0036 (0.0278)	0.0043 (0.0266)	0.0012 (0.0008)	0.0022 (0.0268)	0.0017 (0.0263)
$NV(3S^*, 1, 10, 10)$	0.1975 (0.0838)	0.0029 (0.0185)	0.0036 (0.0175)	0.0012 (0.0006)	0.0028 (0.0178)	0.0023 (0.0172)
$NV(S^*, 1, 20, 20)$	0.1993 (0.0938)	0.0048 (0.0539)	0.0057 (0.0528)	0.0012 (0.0010)	0.0008 (0.0534)	0.0004 (0.0529)
$NV(2S^*, 1, 20, 20)$	0.1988 (0.0794)	0.0041 (0.0243)	0.0049 (0.0235)	0.0012 (0.0007)	0.0016 (0.0240)	0.0011 (0.0236)
$NV(3S^*, 1, 20, 20)$	0.1972 (0.0864)	0.0030 (0.0151)	0.0037 (0.0146)	0.0012 (0.0006)	0.0027 (0.0151)	0.0022 (0.0147)

Table 18 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0124	$\gamma_1(1)$ 0.0052	$\gamma_1(2)$ 0.0024	$\gamma_2(0)$ 0.0277	$\gamma_2(1)$ 0.0139	$\gamma_2(2)$ 0.0076
$RV(5)$	75.9451 (2.3865)	99.3572 (3.9942)						
$RV(300)$	2.5012 (0.4592)	4.8099 (0.9110)						
$RV(900)$	1.2171 (0.3449)	1.8829 (0.6207)						
$RV(1800)$	0.8666 (0.3457)	1.1536 (0.4908)						
$RV_1(5)$	27.3141 (1.7886)	63.2574 (3.4357)						
$RV_1(300)$	0.6412 (0.4165)	0.5738 (0.7486)						
$RV_1(900)$	0.6297 (0.4057)	0.5531 (0.5541)						
$RV_1(1800)$	0.6048 (0.4864)	0.5251 (0.5204)						
$RV(\delta^*)$	1.0604 (0.3841)	1.2729 (0.5324)	0.0132 (0.0004)			0.0282 (0.0013)		
$RV_1(\delta^*)$	0.6255 (0.4374)	0.5751 (0.5565)	0.0132 (0.0004)			0.0282 (0.0013)		
$RV_2(\delta^*)$	0.6195 (0.5017)	0.5847 (0.5868)	0.0132 (0.0004)			0.0282 (0.0013)		
$K^{TH2}(60)$	0.6084 (0.2190)	0.5086 (0.2561)	0.0132 (0.0004)			0.0282 (0.0013)		
$TSRV$	1.0630 (0.1286)	0.9761 (0.1838)	0.0132 (0.0004)			0.0282 (0.0013)		
$NV(S^*, 0)$	1.0031 (0.0970)	0.9349 (0.1591)	0.0116 (0.0003)			0.0262 (0.0009)		
$NV(2S^*, 0)$	0.7770 (0.0901)	0.6850 (0.1212)	0.0117 (0.0003)			0.0265 (0.0009)		
$NV(3S^*, 0)$	0.7110 (0.1007)	0.6121 (0.1220)	0.0118 (0.0003)			0.0267 (0.0009)		
$NV(S^*, 10)$	0.6205 (0.1096)	0.5122 (0.1726)	0.0124 (0.0003)	0.0021 (0.0505)	0.0196 (0.2563)	0.0277 (0.0010)	-0.0378 (4.3119)	0.0660 (8.6657)
$NV(2S^*, 10)$	0.6177 (0.1014)	0.5105 (0.1292)	0.0124 (0.0003)	0.0027 (0.0451)	0.0154 (0.2204)	0.0277 (0.0010)	-0.0105 (4.2469)	0.0605 (8.6644)
$NV(3S^*, 10)$	0.6144 (0.1124)	0.5067 (0.1301)	0.0124 (0.0003)	0.0028 (0.0423)	0.0132 (0.2034)	0.0277 (0.0010)	-0.0217 (4.2449)	0.0401 (8.6963)
$NV(S^*, 20)$	0.6173 (0.1155)	0.5129 (0.1827)	0.0124 (0.0003)	0.0001 (0.1224)	0.0310 (0.5142)	0.0276 (0.0010)	0.6082 (23.4320)	0.0675 (18.2661)
$NV(2S^*, 20)$	0.6166 (0.1050)	0.5104 (0.1328)	0.0124 (0.0003)	0.0015 (0.1170)	0.0245 (0.4605)	0.0277 (0.0010)	0.5896 (23.1766)	0.1123 (18.2085)
$NV(3S^*, 20)$	0.6136 (0.1165)	0.5062 (0.1336)	0.0124 (0.0004)	0.0021 (0.1149)	0.0216 (0.4422)	0.0277 (0.0010)	0.5143 (23.0929)	-0.0272 (18.2696)

Table 19: Monte Carlo simulation results for scenario: **low pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0052	$\gamma_{12}(-1)$ 0.0060	$\gamma_{12}(0)$ 0.0012	$\gamma_{12}(1)$ 0.0018	$\gamma_{12}(2)$ 0.0014
$RC(5)$	2.9108 (1.3973)					
$RC(300)$	0.3669 (0.4472)					
$RC(900)$	0.2727 (0.3452)					
$RC(1800)$	0.2334 (0.3248)					
$RC_{1,1}(5)$	3.6270 (1.5392)					
$RC_{1,1}(300)$	0.1929 (0.4241)					
$RC_{1,1}(900)$	0.1896 (0.3475)					
$RC_{1,1}(1800)$	0.1952 (0.3795)					
$RC(\delta^*)$	0.4896 (0.6255)			0.0007 (0.0003)		
$RC_{1,1}(\delta^*)$	0.1821 (0.5194)			0.0007 (0.0003)		
$RC_{2,2}(\delta^*)$	0.2097 (0.5298)			0.0007 (0.0003)		
HY	5.2817 (1.5308)					
$HY(S^*)$	1.1998 (0.2339)					
$NV(S^*, 1, 0, 0)$	0.2829 (0.1024)			0.0062 (0.0015)		
$NV(2S^*, 1, 0, 0)$	0.2373 (0.0844)			0.0064 (0.0015)		
$NV(3S^*, 1, 0, 0)$	0.2228 (0.0897)			0.0065 (0.0016)		
$NV(S^*, 1, 10, 10)$	0.2012 (0.1078)	0.0020 (0.0143)	0.0025 (0.0150)	0.0012 (0.0030)	0.0039 (0.0133)	0.0036 (0.0157)
$NV(2S^*, 1, 10, 10)$	0.1989 (0.0899)	0.0021 (0.0090)	0.0028 (0.0095)	0.0013 (0.0021)	0.0036 (0.0082)	0.0032 (0.0098)
$NV(3S^*, 1, 10, 10)$	0.1970 (0.0958)	0.0022 (0.0070)	0.0028 (0.0072)	0.0013 (0.0017)	0.0035 (0.0064)	0.0032 (0.0076)
$NV(S^*, 1, 20, 20)$	0.2018 (0.1196)	0.0033 (0.0275)	0.0038 (0.0267)	0.0013 (0.0031)	0.0026 (0.0268)	0.0024 (0.0279)
$NV(2S^*, 1, 20, 20)$	0.1985 (0.0949)	0.0026 (0.0161)	0.0032 (0.0165)	0.0013 (0.0021)	0.0031 (0.0153)	0.0029 (0.0166)
$NV(3S^*, 1, 20, 20)$	0.1964 (0.1006)	0.0023 (0.0131)	0.0029 (0.0131)	0.0013 (0.0017)	0.0034 (0.0125)	0.0032 (0.0133)

Table 19 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0124	$\gamma_1(1)$ 0.0052	$\gamma_1(2)$ 0.0024	$\gamma_2(0)$ 0.0277	$\gamma_2(1)$ 0.0139	$\gamma_2(2)$ 0.0076
$RV(5)$	34.7022 (1.6973)	39.1710 (2.5325)						
$RV(300)$	2.5224 (0.4664)	4.8316 (0.9276)						
$RV(900)$	1.2241 (0.3788)	1.8978 (0.6087)						
$RV(1800)$	0.8697 (0.3663)	1.1103 (0.4702)						
$RV_1(5)$	25.0786 (1.4905)	33.3975 (2.3461)						
$RV_1(300)$	0.6234 (0.4435)	0.5333 (0.8069)						
$RV_1(900)$	0.6064 (0.4276)	0.5367 (0.5254)						
$RV_1(1800)$	0.5964 (0.4675)	0.5489 (0.5141)						
$RV(\delta^*)$	1.0483 (0.3697)	1.3779 (0.5371)	0.0130 (0.0007)			0.0245 (0.0018)		
$RV_1(\delta^*)$	0.6289 (0.4506)	0.5797 (0.5467)	0.0130 (0.0007)			0.0245 (0.0018)		
$RV_2(\delta^*)$	0.6301 (0.4978)	0.5655 (0.6050)	0.0130 (0.0007)			0.0245 (0.0018)		
$K^{TH2}(60)$	0.6120 (0.2379)	0.5094 (0.2730)	0.0130 (0.0007)			0.0245 (0.0018)		
$TSRV$	0.7350 (0.1397)	0.6293 (0.2356)	0.0130 (0.0007)			0.0245 (0.0018)		
$NV(S^*, 0)$	0.7423 (0.1264)	0.6500 (0.2168)	0.0120 (0.0005)			0.0270 (0.0016)		
$NV(2S^*, 0)$	0.6646 (0.1140)	0.5671 (0.1494)	0.0121 (0.0005)			0.0272 (0.0015)		
$NV(3S^*, 0)$	0.6417 (0.1259)	0.5394 (0.1474)	0.0121 (0.0005)			0.0273 (0.0015)		
$NV(S^*, 10)$	0.6191 (0.1473)	0.5212 (0.2464)	0.0123 (0.0006)	0.0019 (0.2187)	-0.0008 (0.1801)	0.0276 (0.0018)	0.0505 (0.9527)	0.0353 (0.8187)
$NV(2S^*, 10)$	0.6129 (0.1291)	0.5135 (0.1623)	0.0124 (0.0006)	0.0048 (0.2199)	0.0011 (0.1807)	0.0277 (0.0016)	0.0334 (0.9009)	0.0444 (0.8012)
$NV(3S^*, 10)$	0.6104 (0.1408)	0.5063 (0.1601)	0.0124 (0.0006)	0.0886 (0.3374)	0.6789 (1.7136)	0.0277 (0.0016)	0.0127 (0.9026)	0.0553 (0.7979)
$NV(S^*, 20)$	0.6191 (0.1473)	0.5212 (0.2464)	0.0123 (0.0006)	0.0019 (0.2187)	-0.0008 (0.1801)	0.0276 (0.0018)	0.0505 (0.9527)	0.0353 (0.8187)
$NV(2S^*, 20)$	0.6129 (0.1291)	0.5135 (0.1623)	0.0124 (0.0006)	0.0048 (0.2199)	0.0011 (0.1807)	0.0277 (0.0016)	0.0334 (0.9009)	0.0444 (0.8012)
$NV(3S^*, 20)$	0.6104 (0.1408)	0.5063 (0.1601)	0.0124 (0.0006)	0.0886 (0.3374)	0.6789 (1.7136)	0.0277 (0.0016)	0.0127 (0.9026)	0.0553 (0.7979)

Table 20: Monte Carlo simulation results for scenario: **low pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ 0.0052	$\gamma_{12}(-1)$ 0.0060	$\gamma_{12}(0)$ 0.0012	$\gamma_{12}(1)$ 0.0018	$\gamma_{12}(2)$ 0.0014
$RC(5)$	0.4928 (0.5517)					
$RC(300)$	0.2549 (0.4729)					
$RC(900)$	0.2218 (0.3512)					
$RC(1800)$	0.1927 (0.3097)					
$RC_{1,1}(5)$	0.8205 (0.8158)					
$RC_{1,1}(300)$	0.1901 (0.4095)					
$RC_{1,1}(900)$	0.1851 (0.3501)					
$RC_{1,1}(1800)$	0.1905 (0.3713)					
$RC(\delta^*)$	0.3114 (0.6165)			0.0004 (0.0006)		
$RC_{1,1}(\delta^*)$	0.1737 (0.5159)			0.0004 (0.0006)		
$RC_{2,2}(\delta^*)$	0.2280 (0.5330)			0.0004 (0.0006)		
HY	1.1437 (0.9060)					
$HY(S^*)$	0.4774 (0.2371)					
$NV(S^*, 1, 0, 0)$	0.2244 (0.1441)			0.0076 (0.0072)		
$NV(2S^*, 1, 0, 0)$	0.2148 (0.1096)			0.0074 (0.0066)		
$NV(3S^*, 1, 0, 0)$	0.2097 (0.1102)			0.0072 (0.0064)		
$NV(S^*, 1, 10, 10)$	0.1997 (0.1577)	0.0015 (0.0355)	0.0038 (0.0376)	0.0013 (0.0154)	0.0018 (0.0378)	0.0024 (0.0457)
$NV(2S^*, 1, 10, 10)$	0.1987 (0.1146)	0.0025 (0.0172)	0.0029 (0.0193)	0.0012 (0.0075)	0.0023 (0.0187)	0.0026 (0.0186)
$NV(3S^*, 1, 10, 10)$	0.1963 (0.1161)	0.0024 (0.0134)	0.0026 (0.0145)	0.0012 (0.0059)	0.0027 (0.0148)	0.0028 (0.0140)
$NV(S^*, 1, 20, 20)$	0.1930 (0.1933)	0.0050 (0.1414)	0.0023 (0.1077)	0.0014 (0.0282)	0.0035 (0.0983)	0.0046 (0.1280)
$NV(2S^*, 1, 20, 20)$	0.1992 (0.1205)	0.0029 (0.0194)	0.0029 (0.0202)	0.0012 (0.0078)	0.0025 (0.0193)	0.0024 (0.0197)
$NV(3S^*, 1, 20, 20)$	0.1963 (0.1211)	0.0024 (0.0144)	0.0028 (0.0144)	0.0012 (0.0059)	0.0029 (0.0148)	0.0026 (0.0143)

Table 20 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **low pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0005	$\gamma_1(1)$ 0.0004	$\gamma_1(2)$ 0.0003	$\gamma_2(0)$ 0.0007	$\gamma_2(1)$ 0.0006	$\gamma_2(2)$ 0.0005
$RV(5)$	3.2583 (0.0742)	4.0554 (0.1093)						
$RV(300)$	0.6948 (0.1142)	0.6137 (0.1042)						
$RV(900)$	0.6252 (0.1717)	0.5257 (0.1440)						
$RV(1800)$	0.5935 (0.2400)	0.4924 (0.1957)						
$RV_1(5)$	2.2620 (0.0939)	2.5927 (0.1127)						
$RV_1(300)$	0.6117 (0.1797)	0.5075 (0.1520)						
$RV_1(900)$	0.6012 (0.2931)	0.4940 (0.2424)						
$RV_1(1800)$	0.5814 (0.4020)	0.4923 (0.3390)						
$RV(\delta^*)$	0.6656 (0.1322)	0.5670 (0.1211)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_1(\delta^*)$	0.5993 (0.1966)	0.4971 (0.1729)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_2(\delta^*)$	0.6089 (0.2357)	0.5055 (0.2108)	0.0013 (0.0001)			0.0013 (0.0001)		
$K^{TH2}(60)$	0.6114 (0.1372)	0.5038 (0.1213)	0.0013 (0.0001)			0.0013 (0.0001)		
$TSRV$	1.1113 (0.1010)	0.8154 (0.0731)	0.0013 (0.0001)			0.0013 (0.0001)		
$NV(S^*, 0)$	1.2188 (0.0452)	0.8501 (0.0429)	0.0002 (0.0000)			0.0004 (0.0000)		
$NV(2S^*, 0)$	0.8795 (0.0451)	0.6496 (0.0438)	0.0003 (0.0000)			0.0005 (0.0000)		
$NV(3S^*, 0)$	0.7783 (0.0497)	0.5936 (0.0490)	0.0003 (0.0000)			0.0005 (0.0000)		
$NV(S^*, 10)$	0.5819 (0.0622)	0.4873 (0.0606)	0.0005 (0.0000)	0.0005 (0.0000)	0.0004 (0.0001)	0.0007 (0.0000)	0.0006 (0.0013)	0.0005 (0.0039)
$NV(2S^*, 10)$	0.6099 (0.0571)	0.5060 (0.0532)	0.0005 (0.0000)	0.0005 (0.0000)	0.0004 (0.0001)	0.0007 (0.0000)	0.0006 (0.0013)	0.0007 (0.0039)
$NV(3S^*, 10)$	0.6155 (0.0607)	0.5088 (0.0584)	0.0005 (0.0000)	0.0005 (0.0000)	0.0004 (0.0001)	0.0007 (0.0000)	0.0006 (0.0013)	0.0006 (0.0039)
$NV(S^*, 20)$	0.5943 (0.0828)	0.5102 (0.0840)	0.0005 (0.0001)	0.0005 (0.0001)	0.0004 (0.0001)	0.0007 (0.0001)	0.0006 (0.0023)	0.0005 (0.0070)
$NV(2S^*, 20)$	0.6206 (0.0639)	0.5120 (0.0582)	0.0005 (0.0000)	0.0004 (0.0000)	0.0004 (0.0001)	0.0007 (0.0001)	0.0006 (0.0022)	0.0005 (0.0069)
$NV(3S^*, 20)$	0.6209 (0.0660)	0.5117 (0.0628)	0.0005 (0.0000)	0.0004 (0.0001)	0.0004 (0.0001)	0.0007 (0.0001)	0.0006 (0.0023)	0.0004 (0.0070)

Table 21: Monte Carlo simulation results for scenario: **high pers., low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0002
$RC(5)$	0.5621 (0.0575)					
$RC(300)$	0.2222 (0.0830)					
$RC(900)$	0.1933 (0.1274)					
$RC(1800)$	0.1861 (0.1703)					
$RC_{1,1}(5)$	0.8769 (0.0711)					
$RC_{1,1}(300)$	0.1950 (0.1287)					
$RC_{1,1}(900)$	0.1939 (0.2060)					
$RC_{1,1}(1800)$	0.1927 (0.2885)					
$RC(\delta^*)$	0.3298 (0.0639)			0.0000 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1976 (0.0683)			0.0000 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1997 (0.0795)			0.0000 (0.0000)		
HY	0.3382 (0.0613)					
$HY(S^*)$	0.3677 (0.0335)					
$NV(S^*, 1, 0, 0)$	0.3102 (0.0340)			0.0000 (0.0000)		
$NV(2S^*, 1, 0, 0)$	0.2473 (0.0356)			0.0001 (0.0000)		
$NV(3S^*, 1, 0, 0)$	0.2281 (0.0396)			0.0001 (0.0000)		
$NV(S^*, 1, 10, 10)$	0.1993 (0.0476)	0.0002 (0.0034)	0.0003 (0.0034)	0.0001 (0.0001)	-0.0001 (0.0035)	0.0001 (0.0037)
$NV(2S^*, 1, 10, 10)$	0.1980 (0.0436)	0.0002 (0.0027)	0.0003 (0.0027)	0.0001 (0.0001)	-0.0001 (0.0027)	0.0000 (0.0028)
$NV(3S^*, 1, 10, 10)$	0.1980 (0.0473)	0.0001 (0.0024)	0.0002 (0.0024)	0.0001 (0.0001)	0.0000 (0.0024)	0.0001 (0.0024)
$NV(S^*, 1, 20, 20)$	0.1966 (0.0652)	-0.0000 (0.0071)	0.0001 (0.0075)	0.0001 (0.0001)	0.0002 (0.0073)	0.0003 (0.0075)
$NV(2S^*, 1, 20, 20)$	0.1977 (0.0477)	0.0001 (0.0050)	0.0002 (0.0051)	0.0001 (0.0001)	0.0000 (0.0051)	0.0001 (0.0051)
$NV(3S^*, 1, 20, 20)$	0.1978 (0.0512)	-0.0001 (0.0042)	-0.0000 (0.0042)	0.0001 (0.0001)	0.0003 (0.0042)	0.0003 (0.0041)

Table 21 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., low var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0005	$\gamma_1(1)$ 0.0004	$\gamma_1(2)$ 0.0003	$\gamma_2(0)$ 0.0007	$\gamma_2(1)$ 0.0006	$\gamma_2(2)$ 0.0005
$RV(5)$	2.7984 (0.0858)	2.7102 (0.1051)						
$RV(300)$	0.6920 (0.1141)	0.6168 (0.1045)						
$RV(900)$	0.6265 (0.1703)	0.5274 (0.1466)						
$RV(1800)$	0.5943 (0.2385)	0.4930 (0.1970)						
$RV_1(5)$	2.1577 (0.0912)	2.1909 (0.1020)						
$RV_1(300)$	0.6124 (0.1795)	0.5099 (0.1529)						
$RV_1(900)$	0.6005 (0.2942)	0.4925 (0.2419)						
$RV_1(1800)$	0.5814 (0.4016)	0.4918 (0.3390)						
$RV(\delta^*)$	0.6640 (0.1320)	0.5715 (0.1224)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_1(\delta^*)$	0.6015 (0.1958)	0.4969 (0.1751)	0.0013 (0.0001)			0.0013 (0.0001)		
$RV_2(\delta^*)$	0.6080 (0.2372)	0.5050 (0.2094)	0.0013 (0.0001)			0.0013 (0.0001)		
$K^{TH2}(60)$	0.6104 (0.1437)	0.5064 (0.1258)	0.0013 (0.0001)			0.0013 (0.0001)		
$TSRV$	0.8767 (0.0740)	0.6301 (0.0607)	0.0013 (0.0001)			0.0013 (0.0001)		
$NV(S^*, 0)$	0.9126 (0.0494)	0.6395 (0.0502)	0.0003 (0.0000)			0.0005 (0.0000)		
$NV(2S^*, 0)$	0.7416 (0.0525)	0.5624 (0.0543)	0.0003 (0.0000)			0.0006 (0.0000)		
$NV(3S^*, 0)$	0.6921 (0.0587)	0.5412 (0.0607)	0.0004 (0.0000)			0.0006 (0.0000)		
$NV(S^*, 10)$	0.6050 (0.0706)	0.5079 (0.0790)	0.0005 (0.0000)	0.0004 (0.0024)	0.0010 (0.0141)	0.0007 (0.0001)	0.0018 (0.1729)	0.0034 (0.2904)
$NV(2S^*, 10)$	0.6176 (0.0659)	0.5104 (0.0678)	0.0005 (0.0000)	0.0006 (0.0025)	0.0012 (0.0134)	0.0007 (0.0001)	0.0014 (0.1540)	0.0014 (0.2961)
$NV(3S^*, 10)$	0.6181 (0.0710)	0.5106 (0.0730)	0.0005 (0.0001)	0.0006 (0.0026)	0.0012 (0.0130)	0.0007 (0.0001)	0.0022 (0.1528)	0.0016 (0.3086)
$NV(S^*, 20)$	0.6135 (0.0935)	0.5154 (0.1692)	0.0005 (0.0001)	-0.0000 (0.0062)	0.0026 (0.0315)	0.0007 (0.0003)	0.0334 (1.1046)	0.0398 (1.0503)
$NV(2S^*, 20)$	0.6206 (0.0707)	0.5113 (0.0736)	0.0005 (0.0001)	-0.0001 (0.0060)	0.0026 (0.0282)	0.0007 (0.0001)	0.0324 (0.9998)	0.0212 (0.7353)
$NV(3S^*, 20)$	0.6193 (0.0752)	0.5109 (0.0782)	0.0005 (0.0001)	-0.0001 (0.0060)	0.0028 (0.0264)	0.0007 (0.0001)	0.0285 (0.9794)	0.0174 (0.7515)

Table 22: Monte Carlo simulation results for scenario: **high pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0002
$RC(5)$	0.2455 (0.0424)					
$RC(300)$	0.2106 (0.0826)					
$RC(900)$	0.1938 (0.1268)					
$RC(1800)$	0.1870 (0.1705)					
$RC_{1,1}(5)$	0.5123 (0.0628)					
$RC_{1,1}(300)$	0.1962 (0.1288)					
$RC_{1,1}(900)$	0.1915 (0.2045)					
$RC_{1,1}(1800)$	0.1935 (0.2876)					
$RC(\delta^*)$	0.2466 (0.0573)			0.0001 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1939 (0.0731)			0.0001 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1956 (0.0913)			0.0001 (0.0000)		
HY	0.4177 (0.0627)					
$HY(S^*)$	0.3217 (0.0389)					
$NV(S^*, 1, 0, 0)$	0.2411 (0.0415)			0.0002 (0.0001)		
$NV(2S^*, 1, 0, 0)$	0.2167 (0.0437)			0.0003 (0.0001)		
$NV(3S^*, 1, 0, 0)$	0.2093 (0.0490)			0.0003 (0.0001)		
$NV(S^*, 1, 10, 10)$	0.1999 (0.0633)	0.0001 (0.0047)	0.0001 (0.0031)	0.0000 (0.0031)	0.0002 (0.0056)	0.0003 (0.0070)
$NV(2S^*, 1, 10, 10)$	0.1986 (0.0516)	0.0001 (0.0016)	0.0001 (0.0017)	0.0001 (0.0004)	0.0001 (0.0015)	0.0001 (0.0019)
$NV(3S^*, 1, 10, 10)$	0.1981 (0.0566)	0.0001 (0.0013)	0.0001 (0.0013)	0.0001 (0.0003)	0.0001 (0.0012)	0.0001 (0.0015)
$NV(S^*, 1, 20, 20)$	0.1999 (0.0641)	0.0000 (0.0068)	-0.0000 (0.0058)	0.0000 (0.0049)	0.0003 (0.0101)	0.0005 (0.0145)
$NV(2S^*, 1, 20, 20)$	0.1982 (0.0581)	0.0000 (0.0037)	0.0001 (0.0035)	0.0001 (0.0005)	0.0001 (0.0038)	0.0002 (0.0036)
$NV(3S^*, 1, 20, 20)$	0.1977 (0.0621)	0.0000 (0.0027)	0.0001 (0.0026)	0.0001 (0.0003)	0.0002 (0.0027)	0.0002 (0.0028)

Table 22 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., low var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0005	$\gamma_1(1)$ 0.0004	$\gamma_1(2)$ 0.0003	$\gamma_2(0)$ 0.0007	$\gamma_2(1)$ 0.0006	$\gamma_2(2)$ 0.0005
$RV(5)$	1.8489 (0.0832)	1.4300 (0.0859)						
$RV(300)$	0.6938 (0.1164)	0.6164 (0.1040)						
$RV(900)$	0.6240 (0.1735)	0.5265 (0.1443)						
$RV(1800)$	0.5936 (0.2436)	0.4909 (0.1971)						
$RV_1(5)$	1.6697 (0.0840)	1.3333 (0.0845)						
$RV_1(300)$	0.6133 (0.1786)	0.5067 (0.1504)						
$RV_1(900)$	0.5981 (0.2988)	0.4918 (0.2423)						
$RV_1(1800)$	0.5798 (0.3989)	0.4901 (0.3363)						
$RV(\delta^*)$	0.6733 (0.1308)	0.5751 (0.1181)	0.0013 (0.0001)			0.0012 (0.0001)		
$RV_1(\delta^*)$	0.6028 (0.2004)	0.4919 (0.1724)	0.0013 (0.0001)			0.0012 (0.0001)		
$RV_2(\delta^*)$	0.6064 (0.2405)	0.5022 (0.2027)	0.0013 (0.0001)			0.0012 (0.0001)		
$K^{TH2}(60)$	0.6131 (0.1545)	0.5055 (0.1329)	0.0013 (0.0001)			0.0012 (0.0001)		
$TSRV$	0.7098 (0.0698)	0.5445 (0.0759)	0.0013 (0.0001)			0.0012 (0.0001)		
$NV(S^*,0)$	0.7371 (0.0609)	0.5673 (0.0754)	0.0004 (0.0000)			0.0006 (0.0001)		
$NV(2S^*,0)$	0.6681 (0.0658)	0.5327 (0.0703)	0.0004 (0.0000)			0.0006 (0.0001)		
$NV(3S^*,0)$	0.6477 (0.0729)	0.5241 (0.0750)	0.0004 (0.0000)			0.0006 (0.0001)		
$NV(S^*,10)$	0.6197 (0.0967)	0.5071 (0.1405)	0.0005 (0.0001)	0.0002 (0.0098)	0.0010 (0.0276)	0.0007 (0.0002)	0.0005 (0.0470)	0.0018 (0.0493)
$NV(2S^*,10)$	0.6203 (0.0838)	0.5103 (0.0869)	0.0005 (0.0001)	0.0002 (0.0135)	0.0007 (0.0118)	0.0007 (0.0002)	0.0021 (0.0480)	0.0009 (0.0357)
$NV(3S^*,10)$	0.6190 (0.0886)	0.5119 (0.0888)	0.0005 (0.0001)	0.0005 (0.0122)	0.0007 (0.0781)	0.0007 (0.0002)	0.0041 (0.0566)	0.0001 (0.0388)
$NV(S^*,20)$	0.6197 (0.0967)	0.5071 (0.1405)	0.0005 (0.0001)	0.0002 (0.0098)	0.0010 (0.0276)	0.0007 (0.0002)	0.0005 (0.0470)	0.0018 (0.0493)
$NV(2S^*,20)$	0.6203 (0.0838)	0.5103 (0.0869)	0.0005 (0.0001)	0.0002 (0.0135)	0.0007 (0.0118)	0.0007 (0.0002)	0.0021 (0.0480)	0.0009 (0.0357)
$NV(3S^*,20)$	0.6190 (0.0886)	0.5119 (0.0888)	0.0005 (0.0001)	0.0005 (0.0122)	0.0007 (0.0781)	0.0007 (0.0002)	0.0041 (0.0566)	0.0001 (0.0388)

Table 23: Monte Carlo simulation results for scenario: **high pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0000	$\gamma_{12}(-1)$ 0.0000	$\gamma_{12}(0)$ 0.0001	$\gamma_{12}(1)$ 0.0002	$\gamma_{12}(2)$ 0.0002
$RC(5)$	0.0458 (0.0250)					
$RC(300)$	0.1838 (0.0803)					
$RC(900)$	0.1836 (0.1255)					
$RC(1800)$	0.1800 (0.1681)					
$RC_{1,1}(5)$	0.1301 (0.0401)					
$RC_{1,1}(300)$	0.1956 (0.1257)					
$RC_{1,1}(900)$	0.1904 (0.2040)					
$RC_{1,1}(1800)$	0.1912 (0.2857)					
$RC(\delta^*)$	0.1621 (0.0594)			0.0001 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1892 (0.0775)			0.0001 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1892 (0.0889)			0.0001 (0.0000)		
HY	0.2678 (0.0585)					
$HY(S^*)$	0.2450 (0.0494)					
$NV(S^*, 1, 0, 0)$	0.2089 (0.0588)			0.0005 (0.0006)		
$NV(2S^*, 1, 0, 0)$	0.2030 (0.0596)			0.0006 (0.0006)		
$NV(3S^*, 1, 0, 0)$	0.2014 (0.0640)			0.0006 (0.0007)		
$NV(S^*, 1, 10, 10)$	0.2078 (0.0588)		0.0014 (0.0122)	0.0001 (0.0024)	-0.0005 (0.0077)	
$NV(2S^*, 1, 10, 10)$	0.1972 (0.0886)	0.0066 (0.1146)	-0.0050 (0.1411)	-0.0007 (0.0387)	0.0027 (0.1550)	0.0042 (0.1026)
$NV(3S^*, 1, 10, 10)$	0.1979 (0.0721)	-0.0003 (0.0065)	0.0002 (0.0053)	-0.0000 (0.0028)	0.0001 (0.0052)	0.0001 (0.0083)
$NV(S^*, 1, 20, 20)$	0.2078 (0.0588)		0.0014 (0.0122)	0.0001 (0.0024)	-0.0005 (0.0077)	
$NV(2S^*, 1, 20, 20)$	0.1972 (0.0886)	0.0066 (0.1146)	-0.0050 (0.1411)	-0.0007 (0.0387)	0.0027 (0.1550)	0.0042 (0.1026)
$NV(3S^*, 1, 20, 20)$	0.2019 (0.4209)	-0.0002 (0.0225)	0.0008 (0.0327)	0.0014 (0.0605)	-0.0024 (0.1326)	-0.0002 (0.0520)

Table 23 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., low var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1	IV^2	$\gamma_1(0)$	$\gamma_1(1)$	$\gamma_1(2)$	$\gamma_2(0)$	$\gamma_2(1)$	$\gamma_2(2)$
	0.6265	0.5148	0.0051	0.0042	0.0034	0.0068	0.0059	0.0051
$RV(5)$	26.9686	35.9252						
	(0.6281)	(0.9896)						
$RV(300)$	1.4043	1.5597						
	(0.2421)	(0.2758)						
$RV(900)$	0.8523	0.8314						
	(0.2410)	(0.2410)						
$RV(1800)$	0.7005	0.6428						
	(0.2794)	(0.2558)						
$RV_1(5)$	17.0155	21.2956						
	(0.7616)	(1.0095)						
$RV_1(300)$	0.6119	0.5201						
	(0.2645)	(0.2804)						
$RV_1(900)$	0.6082	0.5101						
	(0.3303)	(0.3025)						
$RV_1(1800)$	0.5966	0.4990						
	(0.4360)	(0.3729)						
$RV(\delta^*)$	0.8656	0.7939	0.0059			0.0074		
	(0.2621)	(0.2566)	(0.0002)			(0.0002)		
$RV_1(\delta^*)$	0.6198	0.5169	0.0059			0.0074		
	(0.3326)	(0.3037)	(0.0002)			(0.0002)		
$RV_2(\delta^*)$	0.6344	0.5266	0.0059			0.0074		
	(0.3954)	(0.3947)	(0.0002)			(0.0002)		
$K^{TH2}(60)$	0.6123	0.5085	0.0059			0.0074		
	(0.1805)	(0.1705)	(0.0002)			(0.0002)		
$TSRV$	2.6845	1.6933	0.0059			0.0074		
	(0.3075)	(0.1800)	(0.0002)			(0.0002)		
$NV(S^*,0)$	2.8361	1.5742	0.0027			0.0050		
	(0.1121)	(0.0878)	(0.0001)			(0.0001)		
$NV(2S^*,0)$	1.5655	0.9516	0.0031			0.0054		
	(0.0802)	(0.0727)	(0.0001)			(0.0001)		
$NV(3S^*,0)$	1.1954	0.7760	0.0033			0.0056		
	(0.0799)	(0.0766)	(0.0001)			(0.0001)		
$NV(S^*,10)$	0.5089	0.4658	0.0052	0.0046	0.0040	0.0069	0.0061	0.0075
	(0.1206)	(0.0951)	(0.0002)	(0.0002)	(0.0006)	(0.0002)	(0.0111)	(0.0333)
$NV(2S^*,10)$	0.5838	0.4976	0.0052	0.0046	0.0039	0.0068	0.0062	0.0076
	(0.0867)	(0.0795)	(0.0002)	(0.0002)	(0.0006)	(0.0002)	(0.0110)	(0.0322)
$NV(3S^*,10)$	0.5997	0.5036	0.0051	0.0046	0.0039	0.0068	0.0063	0.0071
	(0.0878)	(0.0848)	(0.0002)	(0.0002)	(0.0006)	(0.0002)	(0.0108)	(0.0312)
$NV(S^*,20)$	0.5996	0.5105	0.0051	0.0045	0.0039	0.0068	0.0056	0.0050
	(0.1388)	(0.1061)	(0.0002)	(0.0002)	(0.0007)	(0.0002)	(0.0184)	(0.0549)
$NV(2S^*,20)$	0.6154	0.5120	0.0051	0.0045	0.0039	0.0068	0.0056	0.0049
	(0.0910)	(0.0837)	(0.0002)	(0.0002)	(0.0007)	(0.0002)	(0.0178)	(0.0514)
$NV(3S^*,20)$	0.6169	0.5112	0.0051	0.0045	0.0039	0.0068	0.0056	0.0048
	(0.0916)	(0.0887)	(0.0002)	(0.0002)	(0.0007)	(0.0002)	(0.0174)	(0.0489)

Table 24: Monte Carlo simulation results for scenario: **high pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0004	$\gamma_{12}(-1)$ 0.0002	$\gamma_{12}(0)$ 0.0010	$\gamma_{12}(1)$ 0.0016	$\gamma_{12}(2)$ 0.0020
$RC(5)$	4.6180 (0.5037)					
$RC(300)$	0.4469 (0.1970)					
$RC(900)$	0.2650 (0.1896)					
$RC(1800)$	0.2193 (0.2105)					
$RC_{1,1}(5)$	7.1697 (0.5922)					
$RC_{1,1}(300)$	0.2025 (0.2010)					
$RC_{1,1}(900)$	0.1990 (0.2465)					
$RC_{1,1}(1800)$	0.1932 (0.3131)					
$RC(\delta^*)$	0.6198 (0.2403)			0.0004 (0.0000)		
$RC_{1,1}(\delta^*)$	0.1880 (0.2062)			0.0004 (0.0000)		
$RC_{2,2}(\delta^*)$	0.1944 (0.2166)			0.0004 (0.0000)		
HY	1.5864 (0.5138)					
$HY(S^*)$	1.2558 (0.1276)					
$NV(S^*, 1, 0, 0)$	0.6074 (0.0695)			0.0008 (0.0001)		
$NV(2S^*, 1, 0, 0)$	0.3737 (0.0567)			0.0011 (0.0001)		
$NV(3S^*, 1, 0, 0)$	0.3053 (0.0602)			0.0012 (0.0002)		
$NV(S^*, 1, 10, 10)$	0.1947 (0.0785)	0.0016 (0.0175)	0.0023 (0.0174)	0.0009 (0.0005)	0.0000 (0.0176)	0.0009 (0.0177)
$NV(2S^*, 1, 10, 10)$	0.1978 (0.0628)	0.0012 (0.0120)	0.0019 (0.0118)	0.0010 (0.0003)	0.0005 (0.0120)	0.0012 (0.0118)
$NV(3S^*, 1, 10, 10)$	0.1982 (0.0674)	0.0009 (0.0087)	0.0015 (0.0084)	0.0010 (0.0003)	0.0008 (0.0086)	0.0014 (0.0085)
$NV(S^*, 1, 20, 20)$	0.1937 (0.0865)	-0.0007 (0.0277)	0.0001 (0.0281)	0.0010 (0.0005)	0.0023 (0.0279)	0.0031 (0.0281)
$NV(2S^*, 1, 20, 20)$	0.1977 (0.0670)	-0.0003 (0.0147)	0.0004 (0.0147)	0.0010 (0.0003)	0.0019 (0.0148)	0.0026 (0.0146)
$NV(3S^*, 1, 20, 20)$	0.1980 (0.0710)	-0.0001 (0.0093)	0.0006 (0.0092)	0.0010 (0.0003)	0.0017 (0.0094)	0.0023 (0.0093)

Table 24 ***(cont'd)*:** Monte Carlo simulation results for scenario: **high pers., mod. var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0051	$\gamma_1(1)$ 0.0042	$\gamma_1(2)$ 0.0034	$\gamma_2(0)$ 0.0068	$\gamma_2(1)$ 0.0059	$\gamma_2(2)$ 0.0051
$RV(5)$	22.3787 (0.6979)	22.4495 (0.9026)						
$RV(300)$	1.3881 (0.2476)	1.5712 (0.2864)						
$RV(900)$	0.8557 (0.2399)	0.8387 (0.2439)						
$RV(1800)$	0.7012 (0.2762)	0.6440 (0.2609)						
$RV_1(5)$	15.9612 (0.7103)	17.2612 (0.8564)						
$RV_1(300)$	0.6210 (0.2663)	0.5288 (0.2814)						
$RV_1(900)$	0.6051 (0.3350)	0.5066 (0.3005)						
$RV_1(1800)$	0.5949 (0.4367)	0.4975 (0.3750)						
$RV(\delta^*)$	0.8723 (0.2608)	0.7797 (0.2615)	0.0059 (0.0002)			0.0074 (0.0003)		
$RV_1(\delta^*)$	0.6272 (0.3266)	0.5179 (0.3116)	0.0059 (0.0002)			0.0074 (0.0003)		
$RV_2(\delta^*)$	0.6084 (0.3950)	0.5300 (0.3801)	0.0059 (0.0002)			0.0074 (0.0003)		
$K^{TH2}(60)$	0.6087 (0.1878)	0.5034 (0.1740)	0.0059 (0.0002)			0.0074 (0.0003)		
$TSRV$	1.6653 (0.1925)	0.9513 (0.1263)	0.0059 (0.0002)			0.0074 (0.0003)		
$NV(S^*, 0)$	1.6336 (0.0982)	0.9372 (0.0943)	0.0035 (0.0001)			0.0058 (0.0002)		
$NV(2S^*, 0)$	1.0392 (0.0843)	0.6829 (0.0820)	0.0039 (0.0001)			0.0060 (0.0002)		
$NV(3S^*, 0)$	0.8706 (0.0890)	0.6124 (0.0895)	0.0040 (0.0001)			0.0061 (0.0002)		
$NV(S^*, 10)$	0.5914 (0.1109)	0.5004 (0.1103)	0.0051 (0.0002)	0.0065 (0.0175)	0.0125 (0.0922)	0.0068 (0.0003)	-0.0031 (1.1077)	-0.0287 (2.1338)
$NV(2S^*, 10)$	0.6106 (0.0945)	0.5073 (0.0921)	0.0051 (0.0002)	0.0077 (0.0160)	0.0086 (0.0789)	0.0068 (0.0003)	0.0032 (1.0821)	-0.0320 (2.1603)
$NV(3S^*, 10)$	0.6134 (0.0999)	0.5080 (0.1007)	0.0051 (0.0002)	0.0082 (0.0152)	0.0056 (0.0708)	0.0068 (0.0003)	0.0124 (1.0728)	-0.0143 (2.1734)
$NV(S^*, 20)$	0.6185 (0.1236)	0.5093 (0.1227)	0.0051 (0.0002)	-0.0010 (0.0438)	0.0250 (0.1912)	0.0068 (0.0004)	0.3360 (6.3804)	0.2637 (4.8955)
$NV(2S^*, 20)$	0.6188 (0.0982)	0.5102 (0.0967)	0.0051 (0.0002)	-0.0008 (0.0428)	0.0237 (0.1727)	0.0068 (0.0003)	0.2959 (6.1193)	0.2370 (4.7275)
$NV(3S^*, 20)$	0.6176 (0.1035)	0.5095 (0.1055)	0.0051 (0.0002)	-0.0005 (0.0426)	0.0230 (0.1658)	0.0068 (0.0004)	0.3007 (6.0731)	0.2407 (4.7653)

Table 25: Monte Carlo simulation results for scenario: **high pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0004	$\gamma_{12}(-1)$ 0.0002	$\gamma_{12}(0)$ 0.0010	$\gamma_{12}(1)$ 0.0016	$\gamma_{12}(2)$ 0.0020
$RC(5)$	1.9637 (0.3611)					
$RC(300)$	0.3649 (0.1886)					
$RC(900)$	0.2668 (0.1866)					
$RC(1800)$	0.2240 (0.2113)					
$RC_{1,1}(5)$	4.0162 (0.5000)					
$RC_{1,1}(300)$	0.2015 (0.2031)					
$RC_{1,1}(900)$	0.1921 (0.2428)					
$RC_{1,1}(1800)$	0.1966 (0.3136)					
$RC(\delta^*)$	0.3982 (0.1962)			0.0006 (0.0001)		
$RC_{1,1}(\delta^*)$	0.1951 (0.2035)			0.0006 (0.0001)		
$RC_{2,2}(\delta^*)$	0.1917 (0.2114)			0.0006 (0.0001)		
HY	2.3782 (0.4689)					
$HY(S^*)$	0.9438 (0.1221)					
$NV(S^*, 1, 0, 0)$	0.3528 (0.0736)			0.0030 (0.0006)		
$NV(2S^*, 1, 0, 0)$	0.2658 (0.0663)			0.0033 (0.0006)		
$NV(3S^*, 1, 0, 0)$	0.2405 (0.0723)			0.0035 (0.0006)		
$NV(S^*, 1, 10, 10)$	0.2013 (0.0835)	0.0006 (0.0069)	0.0009 (0.0075)	0.0010 (0.0016)	0.0010 (0.0067)	0.0015 (0.0084)
$NV(2S^*, 1, 10, 10)$	0.1996 (0.0728)	0.0006 (0.0044)	0.0009 (0.0046)	0.0010 (0.0010)	0.0011 (0.0040)	0.0016 (0.0048)
$NV(3S^*, 1, 10, 10)$	0.1988 (0.0791)	0.0004 (0.0035)	0.0008 (0.0037)	0.0010 (0.0008)	0.0012 (0.0033)	0.0017 (0.0038)
$NV(S^*, 1, 20, 20)$	0.1987 (0.0958)	0.0006 (0.0154)	0.0009 (0.0149)	0.0010 (0.0018)	0.0011 (0.0155)	0.0017 (0.0154)
$NV(2S^*, 1, 20, 20)$	0.1981 (0.0782)	0.0001 (0.0081)	0.0006 (0.0078)	0.0010 (0.0010)	0.0016 (0.0078)	0.0021 (0.0082)
$NV(3S^*, 1, 20, 20)$	0.1977 (0.0840)	-0.0002 (0.0063)	0.0003 (0.0062)	0.0010 (0.0008)	0.0019 (0.0061)	0.0024 (0.0064)

Table 25 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., mod. var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0051	$\gamma_1(1)$ 0.0042	$\gamma_1(2)$ 0.0034	$\gamma_2(0)$ 0.0068	$\gamma_2(1)$ 0.0059	$\gamma_2(2)$ 0.0051
$RV(5)$	12.8834 (0.6162)	9.6859 (0.6081)						
$RV(300)$	1.3939 (0.2538)	1.5749 (0.2904)						
$RV(900)$	0.8551 (0.2440)	0.8398 (0.2387)						
$RV(1800)$	0.7015 (0.2899)	0.6349 (0.2577)						
$RV_1(5)$	11.0773 (0.6073)	8.7222 (0.5834)						
$RV_1(300)$	0.6233 (0.2631)	0.5041 (0.2822)						
$RV_1(900)$	0.5991 (0.3473)	0.5016 (0.3037)						
$RV_1(1800)$	0.5900 (0.4224)	0.5002 (0.3682)						
$RV(\delta^*)$	0.8671 (0.2605)	0.8055 (0.2561)	0.0058 (0.0003)			0.0065 (0.0005)		
$RV_1(\delta^*)$	0.6124 (0.3228)	0.5294 (0.3012)	0.0058 (0.0003)			0.0065 (0.0005)		
$RV_2(\delta^*)$	0.6245 (0.4001)	0.5306 (0.3635)	0.0058 (0.0003)			0.0065 (0.0005)		
$K^{TH2}(60)$	0.6150 (0.2019)	0.5055 (0.1852)	0.0058 (0.0003)			0.0065 (0.0005)		
$TSRV$	0.9557 (0.1374)	0.6274 (0.1308)	0.0058 (0.0003)			0.0065 (0.0005)		
$NV(S^*, 0)$	0.9801 (0.1087)	0.6499 (0.1237)	0.0042 (0.0002)			0.0063 (0.0004)		
$NV(2S^*, 0)$	0.7644 (0.0990)	0.5680 (0.1035)	0.0045 (0.0002)			0.0064 (0.0004)		
$NV(3S^*, 0)$	0.7032 (0.1076)	0.5420 (0.1096)	0.0045 (0.0002)			0.0065 (0.0004)		
$NV(S^*, 10)$	0.6152 (0.1377)	0.5104 (0.1545)	0.0051 (0.0003)	0.0007 (0.0837)	0.0193 (0.0767)	0.0068 (0.0006)	0.0362 (0.2633)	0.0007 (0.2124)
$NV(2S^*, 10)$	0.6174 (0.1128)	0.5137 (0.1168)	0.0051 (0.0003)	-0.0017 (0.0857)	0.0038 (0.0740)	0.0068 (0.0005)	0.0529 (0.2532)	-0.0010 (0.2080)
$NV(3S^*, 10)$	0.6155 (0.1218)	0.5089 (0.1225)	0.0051 (0.0003)	-0.0006 (0.0920)	0.0046 (0.0761)	0.0068 (0.0006)	0.0293 (0.2696)	0.0043 (0.2128)
$NV(S^*, 20)$	0.6152 (0.1377)	0.5104 (0.1545)	0.0051 (0.0003)	0.0007 (0.0837)	0.0193 (0.0767)	0.0068 (0.0006)	0.0362 (0.2633)	0.0007 (0.2124)
$NV(2S^*, 20)$	0.6174 (0.1128)	0.5137 (0.1168)	0.0051 (0.0003)	-0.0017 (0.0857)	0.0038 (0.0740)	0.0068 (0.0005)	0.0529 (0.2532)	-0.0010 (0.2080)
$NV(3S^*, 20)$	0.6155 (0.1218)	0.5089 (0.1225)	0.0051 (0.0003)	-0.0006 (0.0920)	0.0046 (0.0761)	0.0068 (0.0006)	0.0293 (0.2696)	0.0043 (0.2128)

Table 26: Monte Carlo simulation results for scenario: **high pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0004	$\gamma_{12}(-1)$ 0.0002	$\gamma_{12}(0)$ 0.0010	$\gamma_{12}(1)$ 0.0016	$\gamma_{12}(2)$ 0.0020
$RC(5)$	0.2796 (0.1801)					
$RC(300)$	0.2372 (0.1879)					
$RC(900)$	0.2045 (0.1853)					
$RC(1800)$	0.1791 (0.2023)					
$RC_{1,1}(5)$	0.8020 (0.2699)					
$RC_{1,1}(300)$	0.1997 (0.1986)					
$RC_{1,1}(900)$	0.1874 (0.2402)					
$RC_{1,1}(1800)$	0.1914 (0.3046)					
$RC(\delta^*)$	0.2798 (0.2072)			0.0004 (0.0002)		
$RC_{1,1}(\delta^*)$	0.1930 (0.2056)			0.0004 (0.0002)		
$RC_{2,2}(\delta^*)$	0.1891 (0.2173)			0.0004 (0.0002)		
HY	0.8762 (0.3045)					
$HY(S^*)$	0.4626 (0.1169)					
$NV(S^*, 1, 0, 0)$	0.2337 (0.0929)			0.0056 (0.0026)		
$NV(2S^*, 1, 0, 0)$	0.2191 (0.0841)			0.0056 (0.0025)		
$NV(3S^*, 1, 0, 0)$	0.2128 (0.0913)			0.0056 (0.0027)		
$NV(S^*, 1, 10, 10)$	0.1984 (0.1105)	0.0022 (0.0404)	-0.0007 (0.0351)	0.0010 (0.0144)	-0.0004 (0.0388)	0.0027 (0.0377)
$NV(2S^*, 1, 10, 10)$	0.1991 (0.0905)	0.0000 (0.0102)	0.0005 (0.0104)	0.0009 (0.0040)	0.0011 (0.0101)	0.0020 (0.0100)
$NV(3S^*, 1, 10, 10)$	0.1977 (0.0974)	0.0000 (0.0073)	0.0007 (0.0076)	0.0009 (0.0031)	0.0013 (0.0076)	0.0019 (0.0073)
$NV(S^*, 1, 20, 20)$	0.1946 (0.3722)	0.0067 (0.5661)	0.0039 (0.3404)	-0.0012 (0.3152)	-0.0035 (0.3408)	-0.0011 (0.3585)
$NV(2S^*, 1, 20, 20)$	0.1987 (0.0973)	-0.0003 (0.0146)	0.0004 (0.0134)	0.0008 (0.0045)	0.0016 (0.0117)	0.0021 (0.0126)
$NV(3S^*, 1, 20, 20)$	0.1971 (0.1031)	-0.0002 (0.0082)	0.0005 (0.0078)	0.0009 (0.0031)	0.0015 (0.0080)	0.0020 (0.0077)

Table 26 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., mod. var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0507	$\gamma_1(1)$ 0.0421	$\gamma_1(2)$ 0.0342	$\gamma_2(0)$ 0.0678	$\gamma_2(1)$ 0.0591	$\gamma_2(2)$ 0.0505
$RV(5)$	264.1064 (6.1632)	354.6324 (9.8324)						
$RV(300)$	8.4938 (1.6277)	11.0051 (2.0319)						
$RV(900)$	3.1095 (1.0100)	3.8966 (1.2501)						
$RV(1800)$	1.7777 (0.7676)	2.1357 (0.9625)						
$RV_1(5)$	164.5970 (7.4382)	208.3338 (10.0281)						
$RV_1(300)$	0.6353 (1.3166)	0.6488 (1.7568)						
$RV_1(900)$	0.6924 (0.8668)	0.6414 (1.0727)						
$RV_1(1800)$	0.7178 (0.8225)	0.5860 (0.8781)						
$RV(\delta^*)$	1.9075 (0.8271)	2.1987 (0.9788)	0.0515 (0.0015)			0.0685 (0.0021)		
$RV_1(\delta^*)$	0.7129 (0.7841)	0.6215 (0.8811)	0.0515 (0.0015)			0.0685 (0.0021)		
$RV_2(\delta^*)$	0.6789 (0.8400)	0.6571 (0.9161)	0.0515 (0.0015)			0.0685 (0.0021)		
$K^{TH2}(60)$	0.6125 (0.3350)	0.5041 (0.3687)	0.0515 (0.0015)			0.0685 (0.0021)		
$TSRV$	11.1448 (0.6320)	7.3675 (0.4565)	0.0515 (0.0015)			0.0685 (0.0021)		
$NV(S^*,0)$	10.5975 (0.4171)	6.0049 (0.3489)	0.0310 (0.0007)			0.0532 (0.0013)		
$NV(2S^*,0)$	4.8978 (0.2204)	2.8119 (0.1919)	0.0343 (0.0008)			0.0559 (0.0013)		
$NV(3S^*,0)$	3.2352 (0.1745)	1.9044 (0.1644)	0.0358 (0.0008)			0.0571 (0.0014)		
$NV(S^*,10)$	0.2147 (0.3494)	0.3240 (0.3140)	0.0517 (0.0015)	0.0458 (0.0019)	0.0393 (0.0059)	0.0684 (0.0019)	0.0610 (0.1090)	0.0793 (0.3196)
$NV(2S^*,10)$	0.4721 (0.1893)	0.4438 (0.1776)	0.0513 (0.0014)	0.0456 (0.0018)	0.0387 (0.0059)	0.0682 (0.0018)	0.0632 (0.1062)	0.0723 (0.3026)
$NV(3S^*,10)$	0.5305 (0.1643)	0.4717 (0.1629)	0.0512 (0.0014)	0.0455 (0.0018)	0.0385 (0.0059)	0.0681 (0.0018)	0.0651 (0.1039)	0.0648 (0.2889)
$NV(S^*,20)$	0.5572 (0.3621)	0.5163 (0.3259)	0.0509 (0.0015)	0.0447 (0.0020)	0.0386 (0.0072)	0.0678 (0.0019)	0.0549 (0.1752)	0.0522 (0.5059)
$NV(2S^*,20)$	0.5968 (0.1858)	0.5101 (0.1804)	0.0508 (0.0014)	0.0447 (0.0019)	0.0386 (0.0072)	0.0678 (0.0018)	0.0545 (0.1661)	0.0523 (0.4580)
$NV(3S^*,20)$	0.6008 (0.1638)	0.5087 (0.1661)	0.0508 (0.0013)	0.0446 (0.0019)	0.0386 (0.0072)	0.0678 (0.0018)	0.0543 (0.1598)	0.0532 (0.4263)

Table 27: Monte Carlo simulation results for scenario: **high pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0045	$\gamma_{12}(-1)$ 0.0017	$\gamma_{12}(0)$ 0.0100	$\gamma_{12}(1)$ 0.0161	$\gamma_{12}(2)$ 0.0200
$RC(5)$	45.1655 (4.9684)					
$RC(300)$	2.6388 (1.3445)					
$RC(900)$	0.9908 (0.8939)					
$RC(1800)$	0.5409 (0.6427)					
$RC_{1,1}(5)$	70.1192 (5.7930)					
$RC_{1,1}(300)$	0.2817 (1.0879)					
$RC_{1,1}(900)$	0.2169 (0.7498)					
$RC_{1,1}(1800)$	0.2002 (0.6211)					
$RC(\delta^*)$	2.2882 (1.2513)			0.0035 (0.0004)		
$RC_{1,1}(\delta^*)$	0.2166 (1.0644)			0.0035 (0.0004)		
$RC_{2,2}(\delta^*)$	0.1988 (1.0212)			0.0035 (0.0004)		
HY	14.0538 (5.0374)					
$HY(S^*)$	7.0151 (0.7391)					
$NV(S^*, 1, 0, 0)$	2.0255 (0.2350)			0.0107 (0.0014)		
$NV(2S^*, 1, 0, 0)$	0.9871 (0.1372)			0.0125 (0.0015)		
$NV(3S^*, 1, 0, 0)$	0.6855 (0.1215)			0.0132 (0.0015)		
$NV(S^*, 1, 10, 10)$	0.1940 (0.2204)	0.0127 (0.1125)	0.0188 (0.1106)	0.0096 (0.0029)	0.0043 (0.1122)	0.0113 (0.1108)
$NV(2S^*, 1, 10, 10)$	0.1979 (0.1334)	0.0058 (0.0589)	0.0116 (0.0566)	0.0097 (0.0020)	0.0113 (0.0580)	0.0177 (0.0568)
$NV(3S^*, 1, 10, 10)$	0.1979 (0.1241)	0.0029 (0.0379)	0.0084 (0.0357)	0.0098 (0.0017)	0.0142 (0.0371)	0.0204 (0.0360)
$NV(S^*, 1, 20, 20)$	0.1939 (0.2381)	-0.0027 (0.1320)	0.0037 (0.1318)	0.0098 (0.0028)	0.0191 (0.1332)	0.0254 (0.1313)
$NV(2S^*, 1, 20, 20)$	0.1964 (0.1370)	-0.0024 (0.0552)	0.0036 (0.0541)	0.0099 (0.0019)	0.0191 (0.0557)	0.0250 (0.0545)
$NV(3S^*, 1, 20, 20)$	0.1966 (0.1269)	-0.0024 (0.0338)	0.0034 (0.0326)	0.0099 (0.0016)	0.0191 (0.0341)	0.0250 (0.0331)

Table 27 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., high var., high int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0507	$\gamma_1(1)$ 0.0421	$\gamma_1(2)$ 0.0342	$\gamma_2(0)$ 0.0678	$\gamma_2(1)$ 0.0591	$\gamma_2(2)$ 0.0505
$RV(5)$	218.2328	219.8133						
	(6.7812)	(8.8827)						
$RV(300)$	8.3576	11.0663						
	(1.6900)	(2.1365)						
$RV(900)$	3.1100	3.9359						
	(1.0412)	(1.2715)						
$RV(1800)$	1.7710	2.1286						
	(0.7733)	(0.9595)						
$RV_1(5)$	154.0219	167.9419						
	(6.8668)	(8.4129)						
$RV_1(300)$	0.7220	0.6978						
	(1.3613)	(1.7975)						
$RV_1(900)$	0.6758	0.6214						
	(0.9289)	(1.1023)						
$RV_1(1800)$	0.7090	0.5832						
	(0.8260)	(0.8930)						
$RV(\delta^*)$	1.9790	2.1960	0.0515			0.0683		
	(0.8805)	(0.9895)	(0.0019)			(0.0031)		
$RV_1(\delta^*)$	0.7295	0.5880	0.0515			0.0683		
	(0.8228)	(0.8749)	(0.0019)			(0.0031)		
$RV_2(\delta^*)$	0.6903	0.6757	0.0515			0.0683		
	(0.8547)	(0.8887)	(0.0019)			(0.0031)		
$K^{TH2}(60)$	0.6111	0.5291	0.0515			0.0683		
	(0.3524)	(0.3864)	(0.0019)			(0.0031)		
$TSRV$	5.9076	3.0324	0.0515			0.0683		
	(0.4524)	(0.3970)	(0.0019)			(0.0031)		
$NV(S^*, 0)$	5.1542	2.6818	0.0383			0.0598		
	(0.3264)	(0.3327)	(0.0011)			(0.0021)		
$NV(2S^*, 0)$	2.5165	1.4025	0.0407			0.0615		
	(0.1899)	(0.1985)	(0.0012)			(0.0021)		
$NV(3S^*, 0)$	1.7642	1.0428	0.0417			0.0623		
	(0.1663)	(0.1752)	(0.0012)			(0.0022)		
$NV(S^*, 10)$	0.5233	0.4631	0.0510	0.0770	0.0855	0.0680	-0.1062	-0.5221
	(0.3039)	(0.3386)	(0.0017)	(0.1530)	(0.7486)	(0.0027)	(10.4170)	(20.5505)
$NV(2S^*, 10)$	0.5822	0.4961	0.0509	0.0868	0.0370	0.0679	0.0479	-0.3317
	(0.1830)	(0.2005)	(0.0016)	(0.1353)	(0.6176)	(0.0026)	(10.2417)	(20.6155)
$NV(3S^*, 10)$	0.5941	0.4996	0.0508	0.0911	0.0107	0.0679	0.0818	-0.2356
	(0.1688)	(0.1808)	(0.0016)	(0.1271)	(0.5663)	(0.0025)	(10.2407)	(20.7667)
$NV(S^*, 20)$	0.6175	0.5034	0.0507	-0.0078	0.2269	0.0678	2.8325	2.2486
	(0.3170)	(0.3580)	(0.0017)	(0.4130)	(1.6579)	(0.0028)	(57.3805)	(44.8859)
$NV(2S^*, 20)$	0.6142	0.5112	0.0507	-0.0059	0.2247	0.0678	2.8956	2.5592
	(0.1839)	(0.2024)	(0.0017)	(0.4053)	(1.5500)	(0.0026)	(56.6119)	(44.4561)
$NV(3S^*, 20)$	0.6117	0.5077	0.0507	-0.0049	0.2236	0.0678	2.8130	2.4076
	(0.1705)	(0.1829)	(0.0017)	(0.4020)	(1.5130)	(0.0025)	(56.3136)	(44.4081)

Table 28: Monte Carlo simulation results for scenario: **high pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0045	$\gamma_{12}(-1)$ 0.0017	$\gamma_{12}(0)$ 0.0100	$\gamma_{12}(1)$ 0.0161	$\gamma_{12}(2)$ 0.0200
$RC(5)$	19.1531 (3.5523)					
$RC(300)$	1.8593 (1.3044)					
$RC(900)$	0.9917 (0.8635)					
$RC(1800)$	0.5765 (0.6591)					
$RC_{1,1}(5)$	39.0624 (4.8522)					
$RC_{1,1}(300)$	0.2518 (1.0855)					
$RC_{1,1}(900)$	0.1880 (0.7515)					
$RC_{1,1}(1800)$	0.2098 (0.6426)					
$RC(\delta^*)$	1.1773 (1.1101)			0.0061 (0.0010)		
$RC_{1,1}(\delta^*)$	0.2657 (0.9250)			0.0061 (0.0010)		
$RC_{2,2}(\delta^*)$	0.1519 (0.9559)			0.0061 (0.0010)		
HY	21.9630 (4.5404)					
$HY(S^*)$	4.9279 (0.6752)					
$NV(S^*, 1, 0, 0)$	0.8979 (0.2323)			0.0332 (0.0053)		
$NV(2S^*, 1, 0, 0)$	0.5206 (0.1479)			0.0352 (0.0053)		
$NV(3S^*, 1, 0, 0)$	0.4081 (0.1350)			0.0358 (0.0053)		
$NV(S^*, 1, 10, 10)$	0.2143 (0.2280)	0.0054 (0.0383)	0.0084 (0.0385)	0.0098 (0.0080)	0.0117 (0.0333)	0.0168 (0.0407)
$NV(2S^*, 1, 10, 10)$	0.2072 (0.1493)	0.0044 (0.0233)	0.0082 (0.0245)	0.0097 (0.0054)	0.0120 (0.0209)	0.0175 (0.0248)
$NV(3S^*, 1, 10, 10)$	0.2026 (0.1389)	0.0042 (0.0178)	0.0080 (0.0189)	0.0097 (0.0045)	0.0123 (0.0162)	0.0178 (0.0192)
$NV(S^*, 1, 20, 20)$	0.2013 (0.2415)	0.0006 (0.0660)	0.0047 (0.0642)	0.0100 (0.0082)	0.0164 (0.0629)	0.0225 (0.0660)
$NV(2S^*, 1, 20, 20)$	0.2003 (0.1510)	-0.0012 (0.0413)	0.0034 (0.0412)	0.0100 (0.0054)	0.0179 (0.0390)	0.0240 (0.0414)
$NV(3S^*, 1, 20, 20)$	0.1976 (0.1411)	-0.0016 (0.0318)	0.0031 (0.0319)	0.0100 (0.0045)	0.0185 (0.0303)	0.0244 (0.0318)

Table 28 ***(cont'd)*****:** Monte Carlo simulation results for scenario: **high pers., high var., mod. int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IV^1 0.6265	IV^2 0.5148	$\gamma_1(0)$ 0.0507	$\gamma_1(1)$ 0.0421	$\gamma_1(2)$ 0.0342	$\gamma_2(0)$ 0.0678	$\gamma_2(1)$ 0.0591	$\gamma_2(2)$ 0.0505
$RV(5)$	123.2767 (5.9606)	92.2454 (5.9117)						
$RV(300)$	8.3643 (1.6849)	11.1319 (2.2405)						
$RV(900)$	3.1501 (1.0459)	3.9713 (1.2812)						
$RV(1800)$	1.7702 (0.8021)	2.0792 (0.9324)						
$RV_1(5)$	105.1704 (5.8479)	82.6148 (5.6481)						
$RV_1(300)$	0.7170 (1.3704)	0.4908 (1.8882)						
$RV_1(900)$	0.6284 (0.9249)	0.5771 (1.1134)						
$RV_1(1800)$	0.6911 (0.7620)	0.6140 (0.8653)						
$RV(\delta^*)$	1.9547 (0.8652)	2.3423 (1.0365)	0.0510 (0.0028)			0.0592 (0.0043)		
$RV_1(\delta^*)$	0.6891 (0.8137)	0.6408 (0.8710)	0.0510 (0.0028)			0.0592 (0.0043)		
$RV_2(\delta^*)$	0.7103 (0.8200)	0.6511 (0.9371)	0.0510 (0.0028)			0.0592 (0.0043)		
$K^{TH2}(60)$	0.6128 (0.3714)	0.5067 (0.4177)	0.0510 (0.0028)			0.0592 (0.0043)		
$TSRV$	2.2643 (0.3698)	1.2044 (0.4704)	0.0510 (0.0028)			0.0592 (0.0043)		
$NV(S^*, 0)$	2.1521 (0.3210)	1.2470 (0.4512)	0.0444 (0.0019)			0.0640 (0.0038)		
$NV(2S^*, 0)$	1.2432 (0.2060)	0.8037 (0.2421)	0.0458 (0.0019)			0.0650 (0.0037)		
$NV(3S^*, 0)$	0.9882 (0.1892)	0.6778 (0.2071)	0.0464 (0.0019)			0.0655 (0.0036)		
$NV(S^*, 10)$	0.6088 (0.3202)	0.5321 (0.4859)	0.0507 (0.0024)	-0.0583 (0.8081)	-0.2623 (0.6365)	0.0678 (0.0043)	0.3823 (2.0497)	-0.0098 (1.8971)
$NV(2S^*, 10)$	0.6117 (0.2051)	0.5137 (0.2485)	0.0507 (0.0023)	-0.0241 (0.7943)	0.0274 (0.6869)	0.0679 (0.0040)	0.3461 (2.0931)	0.0100 (1.8478)
$NV(3S^*, 10)$	0.6104 (0.1928)	0.5048 (0.2156)	0.0507 (0.0023)	0.0047 (1.5503)	0.2544 (8.0699)	0.0679 (0.0039)	0.3194 (2.0629)	0.0241 (1.8394)
$NV(S^*, 20)$	0.6088 (0.3202)	0.5321 (0.4859)	0.0507 (0.0024)	-0.0583 (0.8081)	-0.2623 (0.6365)	0.0678 (0.0043)	0.3823 (2.0497)	-0.0098 (1.8971)
$NV(2S^*, 20)$	0.6117 (0.2051)	0.5137 (0.2485)	0.0507 (0.0023)	-0.0241 (0.7943)	0.0274 (0.6869)	0.0679 (0.0040)	0.3461 (2.0931)	0.0100 (1.8478)
$NV(3S^*, 20)$	0.6104 (0.1928)	0.5048 (0.2156)	0.0507 (0.0023)	0.0047 (1.5503)	0.2544 (8.0699)	0.0679 (0.0039)	0.3194 (2.0629)	0.0241 (1.8394)

Table 29: Monte Carlo simulation results for scenario: **high pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

Model \ True	IC 0.1987	$\gamma_{12}(-2)$ -0.0045	$\gamma_{12}(-1)$ 0.0017	$\gamma_{12}(0)$ 0.0100	$\gamma_{12}(1)$ 0.0161	$\gamma_{12}(2)$ 0.0200
$RC(5)$	2.6244 (1.7306)					
$RC(300)$	0.7830 (1.3528)					
$RC(900)$	0.4228 (0.8769)					
$RC(1800)$	0.1748 (0.6242)					
$RC_{1,1}(5)$	7.5180 (2.5604)					
$RC_{1,1}(300)$	0.2405 (1.1182)					
$RC_{1,1}(900)$	0.1601 (0.7295)					
$RC_{1,1}(1800)$	0.1901 (0.5754)					
$RC(\delta^*)$	0.8692 (1.4079)			0.0035 (0.0019)		
$RC_{1,1}(\delta^*)$	0.2104 (1.1934)			0.0035 (0.0019)		
$RC_{2,2}(\delta^*)$	0.2288 (1.2352)			0.0035 (0.0019)		
HY	6.9322 (2.8042)					
$HY(S^*)$	2.0260 (0.6263)					
$NV(S^*, 1, 0, 0)$	0.4320 (0.2847)			0.0559 (0.0211)		
$NV(2S^*, 1, 0, 0)$	0.3351 (0.1768)			0.0553 (0.0197)		
$NV(3S^*, 1, 0, 0)$	0.3063 (0.1582)			0.0535 (0.0188)		
$NV(S^*, 1, 10, 10)$	0.2113 (0.3045)	-0.0001 (0.0788)	0.0044 (0.0782)	0.0090 (0.0325)	0.0121 (0.0804)	0.0207 (0.0696)
$NV(2S^*, 1, 10, 10)$	0.2035 (0.1836)	0.0004 (0.0397)	0.0052 (0.0421)	0.0096 (0.0187)	0.0141 (0.0449)	0.0213 (0.0402)
$NV(3S^*, 1, 10, 10)$	0.1997 (0.1664)	0.0013 (0.0307)	0.0057 (0.0337)	0.0099 (0.0143)	0.0150 (0.0344)	0.0221 (0.0312)
$NV(S^*, 1, 20, 20)$	0.2011 (0.3441)	0.0001 (0.1485)	0.0050 (0.1085)	0.0101 (0.0384)	0.0160 (0.1002)	0.0215 (0.0891)
$NV(2S^*, 1, 20, 20)$	0.1972 (0.1887)	-0.0017 (0.0413)	0.0028 (0.0415)	0.0100 (0.0186)	0.0171 (0.0452)	0.0227 (0.0417)
$NV(3S^*, 1, 20, 20)$	0.1950 (0.1695)	-0.0021 (0.0316)	0.0024 (0.0332)	0.0101 (0.0143)	0.0176 (0.0335)	0.0240 (0.0313)

Table 29 ***(cont'd)*:** Monte Carlo simulation results for scenario: **high pers., high var., low int.** Each cell entry consists of the mean and the standard deviation (in parentheses) over 1000 Monte Carlo replications. δ^* denotes the optimal sampling frequency as in equation (8). S^* denotes the optimal number of subgrids as in equation (9).

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